ECM3 Teaching Roundtable
THE ART OF COLLABORATION IN TEACHING: A U.S. PERSPECTIVE

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Overview

Recent decades have seen a remarkable increase in interdisciplinary research. For example, mathematics has made possible whole new fields of biology and finance. As a result, there is new interest in interdisciplinary programs at both the undergraduate and the postgraduate level. In the US, where the majority of a mathematics department's students are likely to be from outside the department, growing collaboration in research and postgraduate training has lead to increased expectations that introductory level courses will reflect the fields that require them.

This paper describes some of the issues that are raised by trying to meet these expectations. Although issues are introduced in a US context, the philosophical question raised—how to teach students mathematics when they are going into other fields—is international.

Background

To clarify students' backgrounds and interests for non-US readers, observe that in 1995 (the latest year for which data is available), 6.5% of the enrolments in US mathematics courses were above the calculus level, 36.5% were at the calculus level, and 57% were below the calculus level.[1] Since virtually none of the students below the calculus level go on to study mathematics, and only a tiny proportion of those at the calculus level, the vast majority of a US mathematics department's teaching is of students from other departments. This raises the question of what, and how, they should be taught. Should they be taught the same material and in the same style as the students who may become mathematicians? (Probably not for students in courses below the calculus level.) In addition, who should make the decision? The mathematics department? The students' department? Some combination of the two? In the past, these decisions have largely been made by the mathematics department, often without input from the students' department. However, expectations are currently changing.

As those who have taught in the US will know, there is a further complicating factor, namely that a student's department may not be fixed. Unlike in some countries, a US undergraduate is admitted to a college or university, not to a department. Only later— perhaps at the end of the student's first year—the student chooses a department. Even then the choice may change. Thus, US mathematics departments have often thrown up their hands at the prospect of tailoring their courses to any one field and decided to teach all students in the same way—when possible, as though they were going to be mathematicians. When this is impractical—for example, students taking calculus for business—courses have grown up that are clearly derived from courses for mathematicians, usually

[1] Data from Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, Fall 1995 CBMS Survey, by Don O. Loftsgaarden, Donald C. Rung, Ann E. Watkins. (MAA Reports Number 2).

by removing theoretical material and adding some applications. However, the applications are often contrived because the mathematics that is being taught is not what is being used in the field. In the last couple of decades, there has been little communication between mathematics and other disciplines on issues of teaching, and these courses have drifted far from their moorings. It is time to re-anchor them.

Successful Collaboration

This section outlines two principles that generally characterise successful collaboration, and suggests ways in which they may be implemented.

1. **Collaboration should have an intellectual underpinning.** A desire for collaboration springs from the intellectual links between the fields; courses that are built around these links are likely to be more successful with both students and faculty.

The first stage in building on these links is to find out what they are. In many cases, this may be easier for faculty in other fields than for mathematicians, as they took mathematics as part of their training, but we may not have studied their fields.

Mathematicians sometimes do not know how their subject is used in other fields at an elementary level. (This is in contrast to the research level, where they are likely to be well informed.) These blind spots vary greatly from field to field: most mathematicians are knowledgeable about how mathematics is used in elementary physics, and many are familiar with its uses in engineering. But few mathematicians know what uses an undergraduate business major makes of mathematics, or how mathematics enters the coursework of an undergraduate biology student. In fact many mathematicians believe— especially in the case of undergraduate business programs—that no real use is made of mathematics, and the mathematics requirement simply serves to restrict enrolment to an overcrowded major. In all fields, productive collaboration cannot begin without the identification of intellectual links.

The best way to learn about these links is by personal contact with other faculty and by looking at the work assigned in different courses. Questions that it is helpful to ask in meeting with faculty from other fields include:

- What mathematical concepts and skill are used in your courses? Could you show me examples, in exams, problem sets, or texts, that show how they are used?
- Where do students stumble mathematically in your courses?

Lists of topics are less helpful than specific course materials, as faculty in mathematics and biology, for example, can look at the same list of topics and see completely different things. A mathematician may see the word "derivative" and think first of the best linear approximation to a function, while a biologist may think first of the rate of change of a population. When these two faculty say their students do not know about derivatives, they mean different things. Thus a list is an excellent place to start, but it is important to go beyond that and look at texts and problems.

The real measure of what another department expects a student to know is in the work it assigns. What do they expect a student to know? What understandings are they assuming students bring to their course? In many cases, two departments may agree on mathematical topics, while from the students' point of view, there is no connection between what they have been taught and what they are expected to know. For example, in teaching differentiation, mathematicians may make sure that students know the derivatives of common functions and can, for example, calculate the derivative of $f(x) = \sqrt{x}$ both directly using a difference quotient and as an inverse function. An economist, on the other hand, may expect students to identify marginal and average costs on a graph of total cost against quantity, and observe that where average cost is minimised, marginal and average costs are equal. A population biologist may give a graph of growth rate against population in the shape of an inverted parabola, and expect students to obtain the logistic-shaped graph of population against time, and observe that the maximum growth rate occurs at half the carrying capacity.

The previous examples demonstrate that mathematicians, economists, and biologist sometimes draw on very different skills and understandings. Students who can follow the mathematician's arguments may not be able to follow the others, and vice versa. From the students' point of view, the examples given are very different because the mathematician's depends on formulae, but the others do not. In fact, many students will complain that "there is no function" in the economist's and biologist's problems, and therefore that they cannot do them. If pushed, these students may make up a formula for a function whose graph looks like the one they were given (or get one by regression on a calculator) and use that to do the problem. However, this still leaves them unable to follow a lecture in which an entirely graphical argument is given.

Upon realising the potential chasm between the use of mathematics in various fields, it is easy for faculty to start discussing about what calculus really is and who should teach what. However from the students' point of view, these discussions are irrelevant; they are concerned about being expected to reason in ways that they have never seen and do not understand. In that respect they are correct, and one of the main goals of collaboration in undergraduate education is to remedy this situation.

2. **Each party to a collaboration should make a recognised contribution.** Both components—making a contribution and having it recognised—are important to the long-term survival of any collaboration. However, they are sometimes forgotten if turf is at issue.

In the case of mathematics courses for students of other disciplines, mathematicians often believe their contribution is their knowledge of the theoretical underpinnings of the mathematics being taught. While this is often invaluable, it is sometimes not as important, such as in the teaching of courses below the level of calculus. Occasionally this extensive knowledge of mathematics may even be an impediment. Mathematicians who have thought deeply about a subject may be reluctant to teach it at the level of rigour and abstraction that another field asks. They want to teach mathematics "properly" (meaning rigorously), or not all. This can lead to courses that do not meet the needs of students.

However, mathematicians often overlook their other important contribution: experience and skill in teaching mathematics. We may take for granted our knowledge of students' misconceptions, where they stumble, and what to do about it. But this is essential information for mathematics instructors. Faculty from other fields are likely to be over-ambitious and cover too many topics without enough mathematical depth. They are likely to go slowly at points where it is not necessary and fast where a slow speed would be preferable. (As many of us did when we first taught.) The speed and superficial coverage can lead to poor results.

In addition, faculty in other fields who do not usually teach first year college students are sometimes out of touch with student backgrounds. Since most faculty imagine students to be similar to themselves, but slower, they find it hard to believe that student backgrounds have changed as much as they have. The prospect of going to college with few algebraic skills is incomprehensible to them—although it is happening in many countries. As a result, faculty from other fields may suggest that mathematicians tell high school teachers to fix this, as though all it took was the wave of a wand.

Thus, mathematicians bring to the table two skills that are essential for the construction of good collaborative courses—a clear vision of mathematical structure and skill in teaching mathematics. What the other discipline brings, and mathematicians must recognise, is the understanding of what mathematics is important to that discipline and knowledge of how it is used.

Implementation

The best way to proceed toward collaboration is almost always a local question. However, there is one guideline that, although hardly weighty, can make a substantial difference to the tone and speed of the collaboration.

> **Committees are no substitute for personal contacts.** Formal committee meetings between faculty who do not regularly work together sometimes have the trappings of a medieval battle, with lines and swords drawn, rather than of an intellectual exchange. For a thoughtful exchange, an informal setting may be better.

Committees of course have their place. But they are for formal ratification of procedures, for example, not for exploration and the exchange of ideas. Lunch, dinner, a cup of coffee, or a glass of wine is better for learning the intellectual links between fields, or for brainstorming about the best ways to teach courses.

The greatest practical problem in a collaborative endeavour is usually time. Any way of discovering the intellectual links between fields takes time. The most effective, but one which is unrealistic for many faculty because it is too time-consuming, is team-teaching with faculty from another discipline. Less time-consuming, but still useful, is temporarily exchanging instructors between departments. Despite the administrative problems that such exchanges sometimes generate, they are enormously worthwhile. In some institutions, exchanging graduate student teaching assistants is possible even if exchanging faculty is not. The regular contact between graduate teaching assistants and faculty provides some of the needed flow of ideas and perspectives between departments.

Other ways of finding out what other departments expect include looking at tests and materials, and talking to students. Answering students' questions about courses in another field gives valuable (and sometime disturbing) insights into what they need to know and what they find hard. It is surprising, and sobering, to see how hard students find making connections between what has been taught and the way it is used in another context. However, former students can be an invaluable source of help. Asking them to bring examples from another field showing the mathematics that they were taught, or wished they had been taught, can produce surprisingly helpful responses.

Conclusion

Collaboratively designed courses have the potential to raise the standards in mathematics courses for students in other disciplines. Currently, students meeting an unfamiliar looking piece of mathematics in another field say they have never seen the topic before. The faculty in the other discipline, not knowing what has been taught, are unable to relate the topic to what the student has learned. Thus an opportunity is lost to deepen students' appreciation of mathematics.

With collaboratively designed courses, faculty from other disciplines can regularly reinforce the mathematics that has been learned, as they will know what has been taught, and in what notation, and from what point of view. Besides the practice that this provides and the increased flexibility that should result, this collaboration makes a powerful statement to students. It says that the mathematics is sufficiently important that it is used in their own fields, as well as in mathematics courses. This is a point that will not be lost on today's students.