Intro and Summary of the Limit function

Limit is a function. A function: For every input there can be only one output.

Idea is: As x approaches c, is the function approaching a value?

$x \to c$ is $f(x) \to \mathcal{L}$

Same value if approaching c from the left, right or any approach.

New Notation

$$\lim_{x \to c} f(x) = \mathcal{L}$$

If the value $\mathcal{L}$ (must be a number) exist as $x \to c$ we say that the limit exist at $x = c$.

$\lim_{x \to c} f(x) = ?$

A. If $\mathcal{L}$ exist then the limit exist, written as: $\lim_{x \to c} f(x) = \mathcal{L}$.

Two results

1. If $\lim_{x \to c} f(x) = \mathcal{L} = f(c)$ *then* $f(x)$ is continuous at $x = c$.
2. If $\lim_{x \to c} f(x) = \mathcal{L} \neq f(c)$ *then* $f(x)$ is not continuous but there is a hole.
   If we want to make $f(x)$ to be continuous at $x = c$ then we set $f(c) = \mathcal{L}$.

B. If $\mathcal{L}$ does not exist it is written $\lim_{x \to c} f(x) = DNE$

Two results

1. There is a jump in the function. The function value on the left of c does not equal the function value to the right of c. $\lim_{x \to c-} f(x) = \mathcal{L} \neq M = \lim_{x \to c+} f(x)$

2. There is a vertical asymptote: The function is going to infinity $f(x) \to \pm\infty$
   The book does allow this notation: $\lim_{x \to c} f(x) = \pm\infty$

$\lim_{x \to \infty} f(x) = \mathcal{L}$ As x gets extremely large is the function approaching a value?

Is there a horizontal asymptote?

Two results

1. If $\lim_{x \to \infty} f(x) = \mathcal{L}$ then there is a horizontal asymptote: $y = \mathcal{L}$.
2. If $\lim_{x \to \infty} f(x) = \pm\infty$ then the limit does not exists.
   $f(x) \to \infty$ the function is growing without bounds.
   $f(x) \to -\infty$ the function is decreasing without bounds.

Rule of 4 on Evaluating a limit function. $\lim_{x \to c} f(x) = \mathcal{L}$

Graphically: $x \to c$ No matter how x approaches c the function seems to be approaching the same value. The function is approaching the same value on the right and left of c. You must zoom in very closely if using your calculator.

$x \to \infty$ $\mathcal{L}$ exist if there is a horizontal asymptote.

Numerically: Make a table of values by picking values of x and evaluating the function. At least 6 values so one can see the approach. You may need more if you can't tell or rounding to a certain number of decimals. Remember $x \neq c$.

$x \to c$ Letting values of x be very close to c ie. $\begin{Bmatrix} c + .001 & c + .0001 & c + .00001 \\ c - .001 & c - .0001 & c - .00001 \end{Bmatrix}$

$x \to \infty$ Pick values that are very large ie. 1 million, 1 billion, 1 trillion etc.

Algebraically: Use the limit rules. Usually you have to do some algebra first. See next page for rules.

Words: $x \to \infty$ There is a horizontals asymptote. In story problems: "eventually" or "in the long run" ie. The flies will increase to a max number of flies of 560. $\lim_{t \to \infty} f(t) = 560$. ie. The yam will reach oven temperature. The Ph balance will eventually stabilize to 4.

Properties of Limits use to evaluate a limit function.
Assuming all the limits on the right hand side exist:

1. $\lim_{x\to c} kf(x) = k\lim_{x\to c} f(x)$ where k is a real number.
2. $\lim_{x\to c}\big(f(x)+g(x)\big) = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$
3. $\lim_{x\to c}\big(f(x)g(x)\big) = (\lim_{x\to c} f(x))(\lim_{x\to c} g(x))$
4. $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$ IF $\boldsymbol{\lim_{x\to c} g(x) \neq 0 \; or \; g(x) \nrightarrow \infty}$
   To use this rule the limit of the denominator must approach a non-zero number.

5. $\boldsymbol{\lim_{x\to c} k = k}$
6. $\boldsymbol{\lim_{x\to c} x = c}$
7. $\lim_{x\to c}\big(f(x)\big)^p = (\lim_{x\to c} f(x))^p$ Limits can move in and out of functions.
8. $\lim_{x\to\infty}\frac{1}{x} = 0$ and $\lim_{x\to-\infty}\frac{1}{x} = 0$ and $\lim_{x\to\infty}\frac{1}{x^2} = 0$
9. $\lim_{x\to\infty} e^{-x} = 0$ and $\lim_{x\to-\infty} e^{x} = 0$
10. If $b > 1$, then $\lim_{x\to\infty} b^{-x} = 0$ and $\lim_{x\to-\infty} b^{x} = 0$
11. If $0 < b < 1$ then $\lim_{x\to\infty} b^{x} = 0$ and $\lim_{x\to-\infty} b^{-x} = 0$

Another note: $\lim_{x\to c}\frac{N(x)}{D(x)}$ or $\lim_{x\to\infty}\frac{N(x)}{D(x)}$ for these limits to possibly exist:

1. $D(c) \neq 0$ or $D(c) \nrightarrow \infty$ [Just use the limit rules and evaluate.]

2. If $D(c) = 0$ then $N(c) = 0$ [A hole]
3. If $D \to \infty$
   For type 2 and 3 one needs to do algebra before evaluating the limit. Must change the denominator to be approaching a non- zero value.

To make a function continuous when there is a hole in the graph. Find the limit of the function as x approaches the point of discontinuity. Set the function equal to this value at the point of discontinuity.

Example:

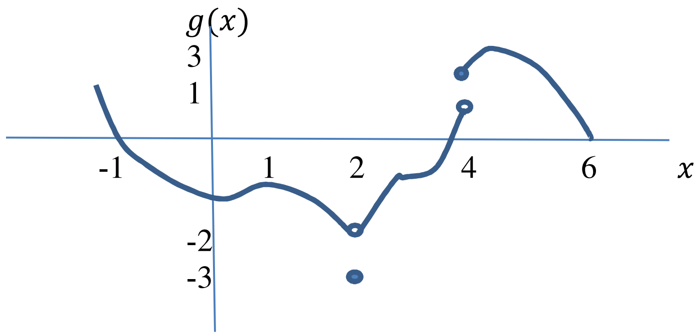
Evaluate $\lim_{h\to 0}\frac{((3+h)^2)-(3^2)}{h} = \lim_{h\to 0}\frac{(9+6h+h^2)-(9)}{h}$

$$= \lim_{h\to 0}\frac{6h+h^2}{h}$$

$$= \lim_{h\to 0}\frac{h(6+h)}{h} = \lim_{h\to 0}(6+h) \overset{eval}{=} 6+0=6$$

Evaluate $\lim_{t\to 5}\frac{t^2-9}{t-5} = DNE$ Since the denominator is approaching zero and the numerator is not. The conclusion at $t = 5$ must be Vertical Asymptotes.

Graphically



a. $\lim_{x\to 1} g(x) =$

b. $\lim_{x\to -1} g(x) =$

c. $\lim_{x\to 2^-} g(x) =$

d. $\lim_{x\to 2^+} g(x) =$

e. $\lim_{x\to 2} g(x) =$

f. $\lim_{x\to 4^-} g(x) =$

g. $\lim_{x\to 4} g(x) =$

h. $g(1) =$

i. $g(2) =$

j. $g(4) =$

Table [For your homework you will be making a table of value – Use evaluate program.]

Use the following results to evaluate $\lim_{x\to 3} T(x) =$____________

x	$T(x)$
2.99	7.4872
2.999	7.4905
2.9999	7.4985
2.99999	7.4999
3.00001	7.4992
3.0001	7.4989
3.001	7.4729

When you make a table you need enough decimal places so you can see it approaching a value. Need 4 or 5 decimals places in the table to approximate 3 decimal places.

Algebraically

Evaluate:

$$\lim_{x\to -3} \frac{x^2+6x+9}{x+3}$$

$$\lim_{x\to \infty} \frac{x^2+6x+9}{3+2x^2}$$