Notation for 1.8

These are the only limits you can evaluate directly. These limits you can just know.

$\lim_{x\to c} k = k$ $\lim_{x\to\infty}\frac{1}{x}=0$ $\lim_{x\to\infty}\frac{1}{x^2}=0$ $\lim_{x\to\infty}\frac{1}{e^x}=0$ $\lim_{x\to-\infty} e^x=0$

$\lim_{x\to c} x = c$ $\lim_{x\to-\infty}\frac{1}{x}=0$ $\lim_{x\to-\infty}\frac{1}{x^2}=0$ $\lim_{x\to-\infty}\frac{1}{e^{-x}}=0$ $\lim_{x\to\infty} e^{-x}=0$

Example: Notation: Before you evaluate any limit function, it must be one from the list above.

$\lim_{h\to o}(4+h)$ *You cannot evaluate at this step.*

You must use the rules of limits before evaluating. Using addition rule of limits

= $\lim_{h\to o}(4)+\lim_{h\to 0}(h)$ Now you can evaluate. When you evaluate the limit, the limit function is gone.

= 4+0 = 4 The limit of 4 as $h\to$ 0 is 4 The word limit is gone.

Also see 4th page of guidelines on homework.

Correct notation: $\lim_{x\to\infty} x^2 = DNE$ reason is as $x\to\infty$ the $x^2\to\infty$ therefore the limit does not exist.

Infinity is not a number. It does not have a number value.
The function can approach infinity but it **does not equal** infinity
For a limit to exist the function must be approaching a number value. **L**
If the limit of a function is going to infinity then the limit does not exist (DNE)

The following is *incorrect notation* $\lim_{x\to\infty} x^2=\infty$ I do not want to see this on you homework

Other information:
Use can use the following facts if needed

Bring limit inside the square root function: $\lim_{h\to 0}\sqrt{4+h}=\sqrt{\lim_{h\to 0}(4+h)}$.

You can also use $\lim_{x\to c} x^2 = \left(\lim_{x\to c} x\right)^2$ but only for x not $f(x)$

The reason for this is that $\lim_{x\to c} x^2 = \left(\lim_{x\to c}(x\cdot x)\right)=\left(\lim_{x\to c} x\right)\left(\lim_{x\to c} x\right)=\left(\lim_{x\to c} x\right)^2$

Hint on #19 make the function a piecewise function by interpreting absolute value function for x>4 and x<4. I have done it in class about $|x|/x$.

Recall: $f(x)=|x|$ you can write as a piecewise function $f(x)=\begin{cases}-x & x<0\\ x & x\geq 0\end{cases}$ now examine each of these functions on its domain.

Finding the limit that is a fraction: *the limit of the denominator must exist and can't equal zero*.
Only can evaluate the limit of $f(x)$ if the limit of the denominator is a non-zero constant.

Evaluate $\lim_{x\to\infty}\frac{N(x)}{D(x)}$

Lets say the limit of the denominator is equal to k $\lim_{x\to a} D(x) = k$ (where k is a non-zero constant)

Then there are four choices

a. Limit of the numerator is zero then $\lim_{x\to\infty}\frac{N(x)}{D(x)} = \frac{0}{k} = 0$

b. Limit of the numerator is a non-zero constant c then $\lim_{x\to\infty}\frac{N(x)}{D(x)} = \frac{c}{k}$

c. As x approaches infinity, the numerator is approaching $\pm\infty$, then $\lim_{x\to\infty}\frac{N(x)}{D(x)} = \frac{DNE}{k} = DNE$

d. If the limit of the numerator DNE, then $\lim_{x\to\infty}\frac{N(x)}{D(x)} = DNE$

Steps $\lim_{x\to\infty}\frac{N(x)}{D(x)}$

1. Reduce fraction
2. Does the limit of the denominator exist?
   The limit of the denominator cannot be approaching infinity, or approaching zero.
   a) if YES go to step 3
   b) if NO; The goal is to force the limit of the denominator to exist
      i) Divide each term by the (lead term/lead coefficient) of the denominator
         example if $D(x) = 3x^2 + 10$ lead term is $3x^2$ but we divide each term by x^2
         if the denominator has e^x in it. $\lim_{x\to\infty} e^x = DNE$ Dividing each term by e^x while $\lim_{x\to\infty}\frac{1}{e^x} = 0$
3. Since the limit of the denominator exist, use the rules of limits to evaluate
4. One of the four choices above should happen.

Example:
What happens if the limit of numerator is going to infinity and how to using the correct notation?

If the limit of the numerator does not exist while the limit of the denominator exists, then the limit of the function does not exist.

Example: Evaluate $\lim_{x\to\infty}\frac{x^3 + 3x^2 - 5}{4x - 3}$

$= \lim_{x\to\infty}\frac{x^2 + 3x^2 - 5\frac{1}{x}}{4 - 3\frac{1}{x}}$ (algebra divide each term by x – the largest power function in denominator)

$= \frac{\left(\lim_{x\to\infty}(x)\right)^2 + 3\lim_{x\to\infty}(x) - 5\lim_{x\to\infty}\left(\frac{1}{x}\right)}{\lim_{x\to\infty}4 - 3\lim_{x\to\infty}\left(\frac{1}{x}\right)}$ (division, addition, and scalar rules of limits)

(Evaluating) $= \frac{(DNE)^2 + 3(DNE) - 5(0)}{4 - 3(0)} = \frac{DNE}{4} = DNE$ (the middle step is optional)