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**Visually Monitoring the 2008 Election**

Every election year, we are bombarded with statements from candidates, campaign policy positions, pundit viewpoints, accusations and responses, and polls, polls, polls. Generally, there is a lot of conflicting information. The 2008 election season was as dizzying as usual, but it was different from previous election years in at least one respect: several websites made pollsters’ data available almost immediately upon its production.

The Statistical Graphics Working Group at Iowa State University, consisting of professors and graduate and undergraduate students with an interest in statistical visualization, explored how to visually display this election data with the goal of helping to sort through information reported in the media. To view their results, visit [www.amstat.org/publications/chance/supplemental](http://www.amstat.org/publications/chance/supplemental).
Fixed-Cost vs. Fixed-Risk Post-Election Audits in Iowa

Jonathan Hobbs, Luke Fostvedt, Adam Pintar, David Rockoff, Eunice Kim, and Randy Griffiths

Electoral integrity has been a forefront issue during the past decade. Policy developments such as voter-verified paper records and post-election audits bolster transparency and voter confidence.

A post-election audit is a manual recount of some or all of the voting units, generally precincts, involved in a particular race. Machine-recorded tallies are typically reported as precinct totals; an audit includes a manual recount of actual ballots to compare to the reported totals. States have a variety of audit laws, many of which mandate recounts of a fixed percentage of randomly selected precincts.

An audit procedure should detect an outcome-altering error with high probability in a cost-effective manner. Audits that limit risk, which is the probability of certifying an incorrect election outcome, achieve these objectives, but can involve complicated procedures.

From a statistical perspective, risk can be thought of as the probability of a type II error failing to detect an incorrect outcome. There is no type I error, or declaring an incorrect outcome when the reported result is correct, because a correct outcome cannot be overturned based on audit results alone.

In races with large margins of victory or many precincts, risk-limiting audits are more cost effective than their fixed-percentage counterparts, and risk-limiting audits can have substantially reduced risk in close races or small populations.

Across the nation, states must balance a number of considerations, including efficiency (cost), statistical effectiveness, public perception, and logistical constraints. Two important logistical constraints governing elections are:

1. That elections are usually administered at the county level. In Iowa, county auditors oversee elections.
2. That vote counts are reported in batches, usually by precinct or polling location. These are the totals audited in a recount.

Iowa’s Proposed Legislation

Optical scan ballots are used in all 99 Iowa counties. As an example, sample ballots from the 2008 general election for Johnson County—Iowa’s fourth-largest county—are available at www.johnson-county.com/auditor/samphal2008general.htm.

A paper record is a prerequisite for a post-election audit, but Iowa is one of 25 states that do not require a statewide manual audit of election results. The state code does include provisions for candidates to request recounts. In addition, county auditors may request administrative recounts in their jurisdictions.

Responding to the concerns of engaged citizens, the Iowa secretary of state formed a post-election task force in 2008. The task force included state legislators, county auditors, and electoral experts. Over the course of a year, the group drafted a bill that was introduced in the 2009 legislative session.

Members of the Iowa task force tackled the challenges for the state, most notably the substantially varying populations across Iowa’s counties. The task force based Iowa’s bill on a post-election audit regulation passed in Minnesota in 2006. The approach attempts to formulate a relatively simple audit procedure that accounts for the state’s uneven spatial distribution of voters.

Specifically, Iowa’s proposed legislation calls for a post-election audit after each general election. Depending on the election year, either the results for president or governor would be audited with another randomly selected state or federal office. Each county would perform its audit by randomly choosing a specified number of precincts and recounting the votes for those precincts. Counties would select one precinct if the county has seven or fewer precincts. Counties with more than seven precincts would select the following:

- Two precincts if the county has 50,000 or fewer registered voters
- Three precincts if the county has 50,001–100,000 registered voters
- Four precincts if the county has more than 100,000 registered voters
Assessing Iowa’s Proposed Legislation

Possible Misscount Situations

There are potentially many situations from apparently benign machine functions to fraudulent actions—that cause misscounts or discrepancies between a precinct’s reported vote totals and an accurate true election outcome. In the instance of a malicious election fraud, a surreptitious strategy for altering the election result is to switch many votes in as few precincts as possible. This vote-switching is perhaps a conservative alternative to a potentially more realistic situation common in U.S. elections in which votes are undercounted, which occurs when the votes tabulated for a particular race are not all recorded as the total ballots cast.

A famous undercount example is the Florida recount in the 2000 presidential election, when a vote for a candidate was not recorded on 178,145 ballots. While a small number of these ballots were individuals who simply chose not to vote for a candidate, there was a large undercount of votes due to a poorly designed butterfly ballot. Further, a Scripps Howard study found such states, as well as 544 individual counties, had a larger percentage of undercounted votes than Florida’s 2.9%. An undercount would effectively have half the impact of a switched vote, so more votes and fewer precincts would need to be changed undercounts than actual vote-switches to alter an election outcome.

Numerical Assessment Procedure

To convey the efficacy of Iowa’s proposed legislation, two mechanisms generating misscounts in a precinct, combined with two dispersal scenarios of misscounted precincts throughout the state. The first dispersal scenario is one of random dispersal. Under this scenario, the precincts in which misscounts are selected randomly with equal probability and without replacement from the state’s precinct population. Such a scenario corresponds to a plausible situation in which the vote tabulating machine malfunctions at random precincts.

The second scenario involves a systematic dispersal of the misscount among all precincts where they are concentrated in the most problematic precincts. This represents a worst-case scenario, since it would take the least amount of misscounted precincts.
to alter the results. This scenario corresponds to a situation in which the election results are being altered by a dishonest entity that wishes to avoid detection (i.e., election fraud).

The two miscount-generating mechanisms are vote-switching and undercounts. To simplify assessments, only two-candidate statewide races are considered—Candidate A vs. Candidate B, where Candidate B is incorrectly reported to have won. The reported election results from the 2006 Iowa general election are used in this assessment.

To assess risk, the first step is to determine the precincts in which miscounts occur. This is accomplished under all combinations of the two miscount-generating mechanisms and two dispersal scenarios by using the idea of maximum within precinct miscount (WPIM), which is taken to be 20%. WPIM is seen as the maximum level of discrepancy that would go unnoticed by observation.

Assuming that the observed, but incorrect, margin of victory (MoV) is the same in all miscounted precincts as in the overall election, a precinct is selected (according to one of the dispersal scenarios) and either 20% of the precinct's total votes are switched from Candidate B to Candidate A or 20% of the precinct's total votes are assumed to be uncounted and should have gone to Candidate A. This is repeated in additional precincts until enough votes have been changed to give Candidate A more than 50% of the total vote.

Under any combination of miscount mechanism and dispersal scenario, there are M counties statewide. For each county \( i = 1, \ldots, M \), let \( k \) denote the number of precincts the county is required to audit, \( n \) the total number of precincts in the county, and \( x \) the number of miscounted precincts in the county. Once the precincts in which miscounts occur have been determined, risk can be computed as the following:

\[
\text{Risk} = \prod_{i=1}^{M} \left[ \frac{P[\text{all miscounted precincts in } i\text{-th county undetected} | x]}{P[\text{all miscounts undetected}]} \right] \\
= \prod_{i=1}^{M} f(0 | n, x, k),
\]

where \( f \) is the hypergeometric probability mass function (pmf). The precinct selection process is a case of sampling without replacement from a dichotomous population, a situation that the hypergeometric distribution describes. This hypergeometric probability will be unity when there are no miscounted precincts in a county, and the probability decreases with an increasing number of precincts with miscounts; more miscounted precincts translates to lower risk. The hypergeometric result is a consequence of the proposed sampling procedure, regardless of how miscounts are dispersed.

**Results**

In the case of random miscount dispersal, there is a distribution of risk because a different random selection of precincts will lead to a different value of risk. The 99th percentile of that distribution is shown in Figure 2, which also shows that the proposed legislation varies in its effectiveness with large discrepancies as the observed MoV decreases. In fact, when the MoV is 0.5% and switches occur in the largest precincts, the risk is more than 70%, versus 20% for randomly occurring switches. At this MoV, the undercount mechanism with random dispersal has a risk of 2%.

For increasing MoV, there is better agreement among all but the most extreme scenario. When MoV exceeds 2%, the risk in the remaining three scenarios is below 0.01 (marked by the horizontal dotted line). However, when random miscounts occur and MoV increases, the sampling procedure becomes increasingly inefficient (in terms of the number of votes counted), since the risk begins to fall far below 0.01. The procedure is inefficient, but has low risk for the undercount scenarios when MoV exceeds 2.0%.

As Figure 2 shows, the vote-switching scenario executed in the largest precincts represents a worst-case scenario.

**A Risk-Limiting Procedure**

As the previous section's numerical assessment illustrates, Iowa's proposed audit procedure would have large risk for small MoV, and the procedure becomes inefficient when the MoV is large. An alternative procedure is a statistical audit with greater efficiency (SAGE). The procedure, which is also known as the
Figure 3. Expected precincts sampled in negative exponential. Horizontal dotted line represents number of precincts sampled under Iowa's proposed legislation.

Figure 4. Median precincts sampled in negative exponential with margin of victory 0.5%
negative exponential (NegExp) method, accounts for varying precinct sizes by assigning a unique probability of audit selection for each precinct. For a race with \( N \) total votes cast and \( n_i \) votes cast in precinct \( i \), the probability \( p_i \) of selecting precinct \( i \) for audit is

\[
p_i = 1 - e^{-n_i \theta(N)}
\]

The desired risk \( r \) is specified beforehand for the entire contest. The quantity \( \epsilon_i \) is an error bound specific to each precinct and reflects the assumed misconduct mechanism. The vote-switching and recount mechanisms are again investigated. Appropriate error-bound values for these two mechanisms, respectively, are

\[
\epsilon_{i, \text{WPM}} = 2\theta(WPM)
\]
\[
\epsilon_{i, \text{MoV}} = \eta_i(WPM)
\]

The NegExp procedure's potential is evaluated with another numerical assessment. For varying MoVs, NegExp is applied for a statewide race with turnout like that of the 2006 general election and WPM of 20%. Figure 3 displays the expected number of precincts sampled under NegExp, with the two misconduct mechanisms compared to that of the fixed count under the proposed legislation. The expected number of precincts can serve as a proxy for the cost of the audit, thus the NegExp procedure has cost that depends on MoV.

The actual NegExp sampling procedure also was simulated county by county for MoVs of 0.5% and 2% under the vote-switching mechanism. As Figure 3 shows, the expected number of precincts sampled statewide is 322 for a 0.5% MoV and 89 for a 2% MoV.

Since elections are administered by county, the breakdown of precincts audited by county is also important. Figures 4 and 5 display the median number of precincts sampled in each county in these two situations. The NegExp procedure would place a larger burden than the proposed legislation on the state's largest counties, most notably Polk County. At the same time, smaller counties would have reduced cost, especially for larger MoV.

**Summary**

Iowa's proposed post-election audit legislation includes provisions for escalation if a county's recount reveals a substantial discrepancy. In addition, the procedure is comprehensive in including all ballots in the selection process.

A numerical assessment finds that the risk may be substantial for close races and the procedure would be inefficient, but have low risk, if the race is not close.

The introduction of comprehensive post-election audit legislation in Iowa is an encouraging development for electoral integrity, as is the recent ASA Board of Directors' recommendation for widespread implementation of risk-limiting audits. We advocate the further development of risk-limiting audits in Iowa and interaction between statisticians and election officials. 

**Further Reading**


