A Short Table of Indefinite Integrals

I. Basic Functions

1. \( \int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1 \)

2. \( \int \frac{1}{x} \, dx = \ln |x| + C \)

3. \( \int a^x \, dx = \frac{1}{\ln a} a^x + C \)

4. \( \int \ln x \, dx = x \ln x - x + C, \quad x > 0 \)

5. \( \int \sin x \, dx = -\cos x + C \)

6. \( \int \cos x \, dx = \sin x + C \)

7. \( \int \tan x \, dx = -\ln |\cos x| + C \)

II. Products of \( e^x, \cos x, \) and \( \sin x \)

8. \( \int e^{ax} \sin(bx) \, dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C \)

9. \( \int e^{ax} \cos(bx) \, dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C \)

10. \( \int \sin(ax) \sin(bx) \, dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b \)

11. \( \int \cos(ax) \cos(bx) \, dx = \frac{1}{a^2 - b^2} [b \cos(ax) \sin(bx) + a \sin(ax) \cos(bx)] + C, \quad a \neq b \)

12. \( \int \sin(ax) \cos(bx) \, dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b \)

III. Product of Polynomial \( p(x) \) with \( \ln x, e^x, \cos x, \) and \( \sin x \)

13. \( \int x^n \ln x \, dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1, \quad x > 0 \)

14. \( \int p(x) e^{ax} \, dx = \frac{1}{a} p(x) e^{ax} - \frac{1}{a} \int p'(x) e^{ax} \, dx \)

\( = \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \ldots \)

\( (+, +, +, +, \ldots) \quad \text{(signs alternate)} \)

15. \( \int p(x) \sin ax \, dx = \frac{1}{a^2} p(x) \cos ax + \frac{1}{a} \int p'(x) \cos ax \, dx \)

\( = \frac{1}{a} p(x) \cos ax + \frac{1}{a^2} p'(x) \sin ax + \frac{1}{a^3} p''(x) \cos ax - \ldots \)

\( (+, +, +, +, \ldots) \quad \text{(signs alternate in pairs after first term)} \)

16. \( \int p(x) \cos ax \, dx = \frac{1}{a^2} p(x) \sin ax - \frac{1}{a} \int p'(x) \sin ax \, dx \)

\( = \frac{1}{a} p(x) \sin ax + \frac{1}{a^2} p'(x) \cos ax - \frac{1}{a^3} p''(x) \sin ax - \ldots \)

\( (+, +, +, +, \ldots) \quad \text{(signs alternate in pairs)} \)
IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x \, dx = \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$, $n$ positive

18. $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$, $n$ positive

19. $\int \frac{1}{\sin^m x} \, dx = -\frac{1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} \, dx$, $m \neq 1$, $m$ positive

20. $\int \frac{1}{\sin x} \, dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$

21. $\int \frac{1}{\cos^m x} \, dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} \, dx$, $m \neq 1$, $m$ positive

22. $\int \frac{1}{\cos x} \, dx = \frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C$

23. $\int \sin^m x \cos^n x \, dx$: If $m$ is odd, let $w = \sin x$. If $n$ is odd, let $w = \sin x$. If both $m$ and $n$ are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If $m$ and $n$ are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both $m$ and $n$ are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \frac{x}{a} + C$, $a \neq 0$

25. $\int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln |x^2 + a^2| + \frac{c}{a} \arctan \frac{x}{a} + C$, $a \neq 0$

26. $\int \frac{1}{(x-a)(x-b)} \, dx = \frac{1}{a-b} \ln |x-a| - \ln |x-b| + C$, $a \neq b$

27. $\int \frac{cx + d}{(x-a)(x-b)} \, dx = \frac{1}{a-b} \left[ (ac + d) \ln |x-a| - (bc + d) \ln |x-b| \right] + C$, $a \neq b$

VI. Integrands Involving $\sqrt{a^2 + x^2}$, $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$, $a > 0$

28. $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \frac{x}{a} + C$

29. $\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln |x + \sqrt{x^2 + a^2}| + C$

30. $\int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left( x \sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C$

31. $\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left( x \sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C$