Math 454, Home Work

(I) Answer the following questions for the equation

$$\frac{dx}{dt} = -x + y + x^2 + xy + x^3, \quad \frac{dy}{dt} = x - y + x^2 + xy + y^3.$$  

(a) Is this a one-zero eigenvalue case at (0,0)?
(b) Use a linear change of variables to turn this equation into standard form.
(c) Calculate the leading term of the center manifold.
(d) Determine the stability of (0,0).

(II) Determine the stability of given equations at (0,0) by first calculate the leading term of the center manifold.

(a) $$\frac{dx}{dt} = -x^2 - y^2, \quad \frac{dy}{dt} = -y.$$
(b) $$\frac{dx}{dt} = x^3 - y^2, \quad \frac{dy}{dt} = -y + x^2.$$
(c) $$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -y + x^2.$$

(III) Show that the equilibrium point at (0,0) of the system

$$\frac{dx}{dt} = xy + ax^3 + bxy^2, \quad \frac{dy}{dt} = -y + cx^2 + dx^2y$$

is (a) asymptotically stable if $$a + c < 0$$, and (b) asymptotically unstable if $$a + c > 0$$.

(IV) Consider the system

$$\frac{dx}{dt} = -x^3, \quad \frac{dy}{dt} = -y + x^2.$$ 

(a) Write $$h(x) = \sum_{n=1}^{\infty} h_n x^n$$. Calculate $$h_n$$ for all $$n$$.
(b) Determine the radius of convergence of $$h(x)$$ obtained in (a).
(c) Is (0,0) a stable or unstable fix point?