PROBLEM SET 1

Let $f(x)$ be a function that is defined on $\mathbb{R}^n$. It is called upper semi-continuous at a point x if for every $\epsilon > 0$ there exists $\delta > 0$ such that $f(y) < f(x) + \epsilon$ as long as $|y - x| < \delta$. A function is called upper semi-continuous if it is upper semi-continuous at every point.

Let $f(x)$ and $g(x)$ be upper semi-continuous functions. For each of the following statements, give a proof or a counter-example.

1. $f(x) + g(x)$ is upper semi-continuous.
2. $f(x) - g(x)$ is upper semi-continuous.
3. $f(x)g(x)$ is upper semi-continuous.
4. The set $\{x : f(x) < 0\}$ is open.
5. The set $\{x : f(x) > 0\}$ is open.
6. The set $\{x : f(x) \leq 0\}$ is closed.
7. The set $\{x : f(x) \geq 0\}$ is closed.
8. Let $K \subset \mathbb{R}^n$ be a compact set. Then $\max_{x \in K} f(x)$ exists.
9. Let $K \subset \mathbb{R}^n$ be a compact set. Then $\min_{x \in K} f(x)$ exists.

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