PROBLEM SET 4

Problem 1

Let $f_n(x)$ be a sequence of measurable (extended) functions on $\mathbb{R}^n$. Prove that the set of all points where $\lim_{n\to\infty} f(x)$ exists is measurable.

Problem 2 (courtesy of Sheldon Axler)

Let $f(x)$ be a finite function on $\mathbb{R}$. Suppose that the derivative, $f'(x)$, exists at every point. Prove that the function $f'(x)$ is measurable.

Problem 3

Let $p : \mathbb{R}^2 \to \mathbb{R}$ be the projection onto the x-axis, $p((x, y)) = x$.
a) Prove that if $X \subset \mathbb{R}$ is a set of measure 0 then $p^{-1}(X)$ is a set of measure 0.
b) Prove that if a set $E \subset \mathbb{R}$ is Lesbegue measurable then $p^{-1}(E)$ is Lesbegue measurable.
c) Is it true that $p(Y)$ is Lesbegue measurable for every Lesbegue measurable set $Y \subset \mathbb{R}^2$? Prove or give a counter-example.

Exercises 28, 29. p.44.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX