PROBLEM SET 5

Problem 1

Evaluate the following limits. Justify all steps.

1)
$$\lim_{n\to\infty}\int_0^\infty \left(1+\frac{x}{n}\right)^{-n}\sin\left(\frac{x}{n}\right)dx.$$

2)
$$\lim_{n\to\infty}\int_0^\infty \left(1+\frac{x}{n}\right)^{-n}x^{-1/n}dx.$$

3)
$$\lim_{\epsilon\to 0^+}\epsilon^{1/2}\int_0^\infty \frac{\arctan x}{x^2+\epsilon}dx.$$

4)
$$\lim_{n\to\infty}\int_0^1 (1+nx^2)(1+x^2)^{-n}dx.$$

Problem 2

Prove that
$$\int_{-\infty}^{\infty} e^{-x^2}\cos(ax)dx = \sqrt{\pi}e^{-a^2/4}$$
by expanding the cosine into its Taylor series and justifying term-by-term integration.