PROBLEM SET 7

Problem 1

a) For what values of α the Lesbegue integral

$$\int_{[1,\infty)} \frac{\sin x}{x^{\alpha}} dm(x)$$

is defined?

b) For what values of α the improper Riemann integral

$$\int_{1}^{\infty} \frac{\sin x}{x^{\alpha}} dx$$

converges?

Problem 2

Let f(x) be a continuously differentiable function on \mathbb{R} , and assume that its derivative belongs to $L^p(\mathbb{R})$ for some p > 1. Prove that

$$|f(x) - f(y)| \le ||f'||_p |x - y|^{(p-1)/p}.$$

Problem 3

Let $f(x) \in L^p(\mathbb{R})$. Prove that the function $g_{\alpha}(x) = f(x)/(1+|x|)^{\alpha}$ belongs to $L^1(\mathbb{R})$ when $\alpha > (p-1)/p$. Give an example that shows that $g_{(p-1)/p}$ is not necessarily integrable.

Prroblems 1, 2, 3 from p.p. 95, 96