1. Let 
\[ c_{n,k} = \frac{1}{(k+1)^\alpha + (n+1)^\beta}, \quad k, n \geq 0, \]
be a double series; here \( \alpha \) and \( \beta \) are real numbers.

a) For what values \( \alpha \) and \( \beta \) the sequence \( c = \{c_{n,k}\} \) belongs to \( l^1 \)?

b) For what values of \( \alpha \) and \( \beta \) the sequence \( c \) belongs to \( l^\infty \)?

2. a) Let \( f(x) \) be a convex (i.e. concave up) function of a real variable \( x \). Prove that for any choice of \( x_1, \ldots, x_n \) and \( t_1, \ldots, t_n \) such that \( t_j \geq 0, 1 \leq j \leq n, \) and \( t_1 + \cdots + t_n = 1, \) the following inequality holds

\[ f(t_1x_1 + \cdots + t_nx_n) \leq t_1f(x_1) + t_nf(x_n). \]  
(1)

*Hint:* Try induction in \( n \).

b) Use inequality (1) to prove *Jensen’s inequality:* if \( f(x) \) is a convex function and \( x(\tau) \) is any function then

\[ f\left( \int_0^1 x(\tau)d\tau \right) \leq \int_0^1 f(x(\tau))d\tau. \]  
(2)

c) Show that, in the case when \( f(x) \) is a concave (i.e. concave down) function, both inequalities (1) and (2) hold, with \( \leq \) replaced by \( \geq \).

Exercises 1.2.15, 1.2.17, 1.2.18–1.2.22, 1.3.42.