Math. 527A
Second homework assignment

1. Let

$$c_{n,k} = \frac{1}{(k+1)^\alpha + (n+1)^\beta}, \quad k, n \geq 0,$$

be a double series; here α and β are real numbers.
a) For what values α and β the sequence $c = \{c_{n,k}\}$ belongs to l^1?
b) For what values of α and β the sequence c belongs to l^∞?

2. a) Let $f(x)$ be a convex (i.e. concave up) function of a real variable x. Prove that for any choice of $x_1, \ldots, x_n$ and $t_1, \ldots, t_n$ such that $t_j \geq 0$, $1 \leq j \leq n$, and $t_1 + \cdots + t_n = 1$, the following inequality holds

$$f(t_1 x_1 + \cdots + t_n x_n) \leq t_1 f(x_1) + t_n f(x_n). \tag{1}$$

Hint: Try induction in n.
b) Use inequality (1) to prove *Jensen's inequality*: if $f(x)$ is a convex function and $x(\tau)$ is any function then

$$f\left(\int_0^1 x(\tau) d\tau\right) \leq \int_0^1 f(x(\tau)) d\tau. \tag{2}$$

c) Show that, in the case when $f(x)$ is a concave (i.e. concave down) function, both inequalities (1) and (2) hold, with $\leq$ replaced by $\geq$.

Exercises 1.2.15, 1.2.17, 1.2.18–1.2.22, 1.3.42.