PROBLEM SET 7

Problem 1

Let X be a topological space. A *(parameterized) curve* in X is a continuous function $\gamma : [0,1] \to X$. The space X is called *path connected* if for every pair of points $x, y \in X$ there exists a curve such that $\gamma(0) = x$ and $\gamma(1) = y$.
a) Prove that if X is path connected then it is connected.
b) Let X be a subspace of $\mathbb{R}^2$ with usual metric topology that is given by

$$X = \{(x,y) : \; x = 0, \; -1 \le y \le 1\} \cup \{(x,y) : \; x > 0, \; y = \sin(1/x)\}.$$

Prove that X is not path connected but it is connected.

Problem 2

Prove that $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$.

Problem 3

Let X and Y be topological spaces, and let $f : X \to Y$ be a continuous, one-to-one function onto (injective and surjective). Is it true that the inverse function f^{-1} is necessarily continuous? Prove or give a counterexample.

Exercises 3.2.28, 3.2.43, 3.2.44, 3.2.45, 3.2.49