as a parametric solution of (10). Hence from (9), taking the plus sign before \( \alpha \),
\[
a_1 = 7m^2 + 13mn - 30n^2.
\]
Then from (8), \( a_2 = 13m^2 - 22mn - 26n^2 \). Finally from (5),
\[
a_3 = -8m^2 + 39mn - 16n^2, \quad b_3 = -13m^2 + 24mn - 26n^2.
\]
The negative sign before \( \alpha \) only interchanges \( a_3 \) and \( a_5 \) with sign changed. If we denote the quadratic form \( am^2 + bmn + cn^2 \) by the notation \([a, b, c]\), we write the solution of the system (3) as
\[
\begin{align*}
a_1 &= [7, 13, -30], & a_2 &= [13, -22, -26], & a_3 &= [-8, 39, -16] \\
b_1 &= [-7, 13, -16], & b_2 &= [8, -13, -30], & b_3 &= [-13, 24, -26].
\end{align*}
\]
By Theorem 3, the system (2) has then the following parametric solution:
\[
\begin{align*}
A_1 &= [-7, 62, -30], & A_2 &= [7, 38, -50], & A_3 &= [5, -8, -22], \\
A_4 &= [19, -32, -42], & A_5 &= [-19, 36, -62], & B_1 &= [-9, 66, -42], \\
B_2 &= [5, 42, -62], & B_3 &= [-21, 38, -22], & B_4 &= [9, -14, -50], \\
B_5 &= [21, -36, -30].
\end{align*}
\]
References
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PROJECTING \( m \) ONTO \( \ell_0 \)

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It is a well-known result, due to Phillips, that the Banach space \( m \), of bounded sequences with the sup norm, cannot be projected continuously onto the subspace \( \ell_0 \) of sequences converging to zero [1, page 33, Corollary 4]. A typical use of this fact is found in [2]. We give a simple proof using an idea inherent in [4] and, as was pointed out by the referee, in [3]. Our method may also be used to simplify the proof of the result in [4].

**Lemma [5, page 77].** Let \( I \) be a countable set. Then there is a family \( \{U_a: a \in A\} \) of subsets of \( I \) such that (1) \( U_a \) is infinite, (2) \( U_a \cap U_b \) is finite for \( a \neq b \) and (3) the index set \( A \) is uncountable.

**Proof.** Arthur Kruse has given the following elegant proof: Take \( I \) to be the rationals in \((0, 1)\), \( A \) the irrationals in \((0, 1)\) and, for \( a \) in \( A \), let \( U_a \) be a sequence of rationals in \((0, 1)\) converging to \( a \).

Recall that a subset of the conjugate space \( X^* \) of a Banach space \( X \) is total if the only vector annihilated by all members of the subset is the zero vector.
For brevity we say that a Banach space $X$ has (property) $B$ if $X^*$ contains a countable total subset. It is easy to see that $B$ is preserved under isomorphism, that a subspace of a space with $B$ has $B$ and that the space $m$ has $B$.

**Theorem.** There is no continuous projection of $m$ onto $c_0$.

**Proof.** Suppose that there is a continuous projection of $m$ onto $c_0$. Then $m = c_0 \oplus R$, where $R$ is a closed subspace of $m$. Since $m/c_0$ is isomorphic to $R$ we see that $m/c_0$ has $B$. The proof consists of showing that $m/c_0$ does not have $B$.

We think of $m$ as $B(I)$, the bounded functions on a countable set $I$. Let \( \{ U_a : a \in A \} \) be a family of subsets of $I$ as in the lemma and let $f_a$ be the coset in $m/c_0$ which contains the characteristic function of the set $U_a$.

Let $g$ be in $(m/c_0)^*$. We will show that the set \( \{ f_a : g(f_a) \neq 0 \} \) is countable; it suffices to show that the set $C(n) = \{ f_a : |g(f_a)| \geq 1/n \}$ is countable for each natural number $n$. Choose $f_1, \ldots, f_m$ in $C(n)$ and let $b_i = \text{sgn}(g(f_i)) g(f_i) / |g(f_i)|$. The vector $x = \sum b_i f_i$ is of norm one (note that as a coset $x$ contains vectors whose norm may be greater than one), and so $\|g\| \geq |g(x)| \geq m/n$; thus $C(n)$ is finite for each $n$.

We conclude by noting that if $\{ h_i \}$ is a countable subset of $(m/c_0)^*$ then our argument shows that there are only countably many $f_a$ with $h_i(f_a)$ nonzero for some $i$. Hence we can find a vector $f_a$ which is mapped into zero by all the $h_i$, and so the set $\{ h_i \}$ is not total.

**References**


**INTERIORITY AND THE TONELLI CONDITIONS**

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In 1937, S. Stoilow proved that if $f$ is a complex-valued function of a complex variable which has the properties: (i) point inverses are totally disconnected, and (ii) $f$ maps interior points of its domain of definition into interior points of the image, then $f$ is topologically equivalent to an analytic function. This result stimulated interest in light interior functions (i.e. functions satisfying (i) and (ii)) and in establishing conditions which insure that a function satisfying these conditions will be light and interior. Titus and Young proved that if $f \in C$ and