Superfluorescence pulse shape

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It is shown that the superfluorescence pulse in a laser without mirrors has a universal self-similar shape, which is determined by a single parameter: the pulse delay time.

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The purpose of this paper is to construct a theory of the nonlinear stage of superfluorescence when this phenomenon can be described by one-dimensional semiclassical Maxwell-Bloch equations, ignoring in so doing homogeneous and inhomogeneous line broadening. This theory can be applied, for example, to explain experiments on observation of superfluorescence in beams of cesium atoms.[1]

We shall show that the development of quantum fluctuations in an unstable population-inverted medium results in the formation of a pulse with universal shape, depending in a self-similar manner on the dimensions of the specimen and determined by a single parameter: the individual delay time τ_d. We shall examine superfluorescence in a cylindrical specimen of length L with transverse cross section S, satisfying the condition $F = S/\lambda L \simeq 1$ (F is the Fresnel number, λ is the wavelength). Atoms with dipole moment d and density n such that $nS\lambda \gg 1$, but $(2\pi d^2 n/\hbar\omega) \ll 1$ (here ω is the transition frequency) are distributed in the specimen. In an infinite, population-inverted medium, an instability with increment $\gamma = (2\pi d^2 n\omega/\hbar)^{1/2}$ would develop. In a specimen of finite length $(\gamma L/c) < \sqrt{\ln N}$ (N is the total number of active atoms), after some delay, a pulse forms with characteristic duration τ_0 of the order of the superfluorescence time $\tau_0 \sim \tau_{SF}$ ($\tau_{SF} = c/\gamma^2 L$). The average delay time is (see Refs. 2 and 3) $\langle \tau_d \rangle = (\tau_{SF}/16)\ln^2 N$. We shall asssume that this time is much shorter than the relaxation times T_1 and T_2 and the inhomogeneous line broadening time T_2^*. In experiments with cesium atoms[1] $\langle \tau_d \rangle = 10$ ns, $T_1 = 70$ ns, $T_2 = 80$ ns, and $T_2^* = 32$ ns.

Under the assumptions formulated above, the Maxwell-Bloch equations can be reduced to equations for the complex envelopes of waves traveling in different directions. However, to understand the physical picture of the phenomenon, we shall show that it is sufficient to examine only waves traveling in a single direction. In this case the medium is described by equations for dimensionless envelopes of the electric field $E(x,t)$, population $Z(x,t)$ and polarization $R(x,t)$,

$$\frac{1}{v}\frac{\partial E}{\partial x} + \frac{\partial E}{\partial x} = R, \qquad \frac{\partial R}{\partial t} = EZ,$$

$$\frac{\partial Z}{\partial t} = -\frac{1}{2}(ER^* + E^*R), \qquad v = c\tau_{SF}/L \tag{1}$$

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(we shall use the normalization adopted in Ref. 2). Equations (1) are defined on the segment $0<x<1$ with boundary condition $E(0,t)=0$ and initial conditions

$$E(x,0)=0, \quad R(x,0)=\rho(x), \quad Z(x,0)=(1-|\rho(x)|^2)^{1/2} \tag{2}$$

$\rho(x)$, a Gaussian random function, is determined by the correlation function

$$<\rho(x), \quad \rho^*(x')>=\frac{4}{N}\delta(x-x'). \tag{3}$$

The solution of the linearized ($|E|\ll 1, |R|\ll 1, |Z-1|\ll 1$) equations (1) with initial conditions (2) has the form

$$E(x,t)=\int_0^x dx'\,\rho(x')\theta(-x'+vt)I_0(2\sqrt{(x-x')(t-x/v+x'v)}). \tag{4}$$

Here $I_0(\xi)$ is the Bessel function of imaginary argument and $\theta(x)$ is the Heaviside function.

It is known (see Refs. 4–6) that Eqs. (1) admit a self-similar formulation

$$E=x\,\mathscr{E}(\xi), \quad R=R(\xi), \quad Z=Z(\xi), \quad \xi=2\sqrt{x(t-x/v)}, \tag{5}$$

where $\mathscr{E}(\xi), R(\xi), Z(\xi)$ is the single-parameter family of regular solutions of the system of equations

$$\xi\mathscr{E}_\xi+2\mathscr{E}=2R, \quad 2R_\xi=\xi\mathscr{E}Z, \quad 2Z_\xi=-\frac{\xi}{2}(\mathscr{E}R^*+\mathscr{E}^*R) \tag{6}$$

which depend on the parameter $\mathscr{E}_0=\mathscr{E}|_{\xi=0}$.

Our basic result is the assertion that the solution of the linear problem (4) is continued into the nonlinear region by the self-similar solution (5). Indeed, examining the asymptotic behavior of the solution of the linearized problem with $vt\gg 1$

$$E(x,t)=\frac{2\rho(0)x}{\xi}I_1(\xi)\left[1+0\left(max\left\{\frac{1}{v}\left(\frac{\tau_{SF}}{t}\right)^{1/2}, \quad \frac{\rho'(0)}{\rho(0)}\left(\frac{\tau_{SF}}{\rho t}\right)^{3/4}\right\}\right)\right] \tag{7}$$

[here $\rho(0)\sim N^{-1/2}$ is the value of the initial polarization at the edge of the object] for sufficiently long times ($t>t_{\min}; t_{\min}=\max\{\tau_{SF}, c^2/L^2\tau_{SF}\}$), we verify that the main term in the expansion is purely self-similar. Setting in (7) $\xi=0$, we find that the value of the parameter $\mathscr{E}_0=\rho(0)$.

The self-similar solution for different values of $\mathscr{E}_0$ can easily be found by solving the ordinary equations (6) on a computer. This solution has a first maximum at $t\sim\tau_d\simeq(\tau_{SF}/4)\ln^2(1/|\rho(0)|)$. From the requirement that $\tau_d\gg t_{\min}$, we have the condition on the length of the specimen, for which joining of the linear solution and the self-similar solution occurs for all x,

$$2\gamma L/c\ll\sqrt{\ln N}. \tag{8}$$

We compared the self-similar pulse with the pulse observed experimentally.[1] The parameter $\mathscr{E}_0$ can be determined from the experimentally observed delay time of the first pulse τ_d according to the equation

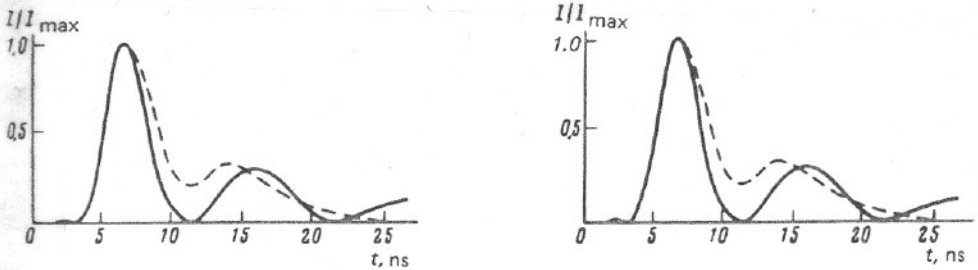

FIG. 1. The solid line is the self-similar solution with $\mathscr{E}_0 = 0.015, \tau_{SF} = 0.5$ ns. The dashed line corresponds to the experiment in Ref. 1.

$$|\mathscr{E}_0|^2 \simeq 4\pi^3\left(\frac{\tau_d}{\tau_{SF}}\right)^{1/2}\exp\left(-4\left(\frac{\tau_d}{\tau_{SF}}\right)^{1/2}\right)$$

derived with logarithmic accuracy. The ratio of parameters $\tau_d/t_{\min}$ was of the order of 20 and, in this case, the shape of the computed pulse exhibited good agreement with the experimental shape (Fig. 1) in the region of the first maximum. The increasing disagreement for large t is apparently related to ignoring inhomogeneous line broadening and diffraction losses.

We have thus established that under our assumptions the development of the superfluorescence process results in the formation of a self-similar pulse with some (small) value of the parameter $\mathscr{E}_0$, determined by fluctuations and polarizations at one end of the specimen.

For any t, each value of the self-similar variable corresponds to two values of the coordinate x. For $t \to \infty$, one of them propagates forward with velocity approaching the velocity of light and the other propagates backwards according to the law $x = \xi_0/4t$, so that the self-similar solution describes an "unloading wave," whose leading and trailing edges move in different directions. Under our assumption (8) concerning the length of the specimen, only the trailing edge, whose group velocity is opposite to the phase velocity, fits within the length of the specimen. In this sense, the situation in the case of superfluorescence is opposite to the situation in an amplifier,[7] where a quasi-self-similar solution also forms, but the main role is played by its leading edge.

The latter circumstance explains the possibility of using waves traveling in one direction. With simultaneous "illumination" of the specimen from both ends, counter-moving unloading waves form. These waves interact weakly until the regions in which the population of the medium changes sign collide. The radiation from the specimen then rapidly ceases. We also note that simultaneous ignition from both ends is quite unlikely, since the dispersion of an individual delay time (see Ref. 2) is of the same order of magnitude as this time.

The result that the superfluorescent pulse has a self-similar nature can be additionally justified with the help of the method of the inverse scattering problem (MISP). The scheme of MISP, which is capable of solving the mixed boundary value problem

for superfluorescence even when inhomogeneous broadening is taken into account, will be published elsewhere.

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Hydrogen and deuterium atoms, stabilized by condensation of an atomic beam in superfluid helium

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Concentrations of H and D atoms, stabilized in a molecular matrix, up to 1×10^{20} cm^{-3} are achieved by condensation of an atomic beam in superfluid helium. Transformation of D atoms into H atoms is observed, indicating the occurrence of tunneling chemical exchange reactions between atomic and molecular hydrogen isotopes.

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Collective quantum effects must be manifested most strongly for hydrogen atoms. For this reason, the problem of accumulating high concentrations of $H(n_H)$ at low temperatures is currently being persistently studied. Work on stabilizing H atoms in the gas phase, spin-oriented in strong magnetic fields ($B\cong11T$ for $T\lesssim0.3$ K), has been very successful.[1] However, only concentrations of 10^{17} cm^{-3} have now been achieved and for a number of reasons it is difficult to expect a large improvement.

For the more traditional method of stabilizing hydrogen atoms in a solid molecular matrix, the most important factor restricting n_H to the 10^{18} cm^{-3} level is the instability of a finite specimen relative to thermal explosion, initiated by the slow, "dark" recombination of atoms.[2] For this reason, when H atoms are frozen out of a

