DEFECTS OF ONE - DIMENSIONAL VORTEX LATTICES

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Abstract

In the framework of the equation

$$\psi_t = \psi_{xx} + \psi - |\psi|^2 \psi \,,$$

the dynamics of one-dimensional lattices of Taylor vortices in Couette flow and of rolls in weak supercritical convection is studied. It is shown that the propagation of the defects as transition areas between stable (according to Eckhaus) and unstable lattices depends significantly on the topological properties of the field $\psi(x)$, i.e. the degree of mapping $R^1 \rightarrow S^1$. The velocity of such defects has been determined. It has been clarified that the defects between stable lattices spread diffusively due to the conservation of the topological invariant.

1. INTRODUCTION

This article studies the dynamics of defects described by the equation

$$\psi_t = \psi_{xx} + (1 - |\psi|^2)\psi \,. \tag{1}$$

This equation arises when we study the weak modulation of a $1D$ vortex lattice over x in the vicinity of the instability threshold in systems whose laminar state had the translational symmetry along some axis. Such systems comprise a flow between two coaxial cylinders with a fixed external and a rotating internal, Couette flow, losing their stability at a certain critical Reynolds number Re, which results in the formation of Taylor vortices, and a flow emerging beyond the threshold of convective instability

of the liquid between two horizontal rigid planes. In these cases the laminar state, i.e., the Couette flow or the stationary heat transfer due to the heat conductivity from lower plane to the upper, is invariant with respect to spatial shift. Therefore, in the vicinity of the threshold the instability growth rate γ_k, dependent on the wave number k, will have, at a certain $k = k_0$, a maximum whose value for a weak supercriticality $\epsilon = (Re - Re_c)/Re_c$ is proportional to ϵ. In the vicinity of the maximum in this case the growth rate can be approximated by the quadratic dependence

$$\gamma_k = \gamma_0 - \alpha(k - k_0)^2, \qquad \gamma_0 \sim \epsilon.$$

This formula shows that at the linear stage a whole range of perturbations with small width $\Delta k \sim \sqrt{\gamma_0} \ll k_0$ is excited. Therefore, to find out the structure of the nonlinear term leading to the saturation of the instability, it is sufficient to average the original equations of motion over "fast" spatial oscillations. This averaging, after some simple rescaling, leads to Eq. (1) for the amplitude. (This is how this equation was derived by Newell and Whitehead [1] for the weak supercritical convection.) Subsequently, the same equation was obtained for the description of a modulation of a $1D$ chain of Taylor vortices near the instability threshold for the Couette flow [2].

It is well known that Eq. (1) can be represented in the variational form as

$$\partial\psi/\partial t = -\delta F/\delta\psi^*, \tag{2}$$

where $F = \int \mathcal{F}dx$ has the meaning of the free energy, with density

$$\mathcal{F} = -|\psi|^2 + |\psi|^4/2 + |\psi_x|^2.$$

The stationary stable state corresponds to the free energy minimum. From (2) it follows that

$$\partial F/\partial t = -2\int |\delta F/\delta\psi|^2 dx < 0. \tag{3}$$

Hence, it is clear that F is a Lyapunov functional, and the state corresponding to the global minimum will, according to the Lyapunov theorem, be stable. To find the stationary point of the functional F it is essential to know the boundary conditions. If one studies (1) on the whole axis $(-\infty < x < +\infty)$, the expression for free energy for distributions nonvanishing at infinity since, only such distributions make physical sense, will linearly diverge with an increasing size. Therefore, in this case in order to define the minimum of F it is sufficient to compare the free energy densities $\mathcal{F}$. If we consider the simplest stationary solutions of (1) in the form

$$\psi = (1 - k^2)^{1/2}e^{ikx}, \qquad 0 < k^2 < 1, \tag{4}$$

the free energy density

$$\mathcal{F} = -(1 - k^2)^2/2,$$

will evidently be minimal at $k = 0$. In this state, from a class of functions nonincreasing at infinity, F will have a global minimum and, therefore, it is stable with respect

to not only small but also finite perturbations. Nevertheless, apart from the solution $\psi = 1$ there exist other stable solutions realizing local minima of F. The study of the stability problem with respect to small perturbations leads to the so-called Eckhaus criterion [3]

$$k^2 \leq 1/3, \qquad (5)$$

of the stability region of solutions (4). It is important to emphasize that the analysis of the stability of arbitrary stationary solutions of (1) showed that there are no other stable solutions except those described above [4].

In the case when the region is finite, which is actually realized in experiments, the question of defining stable solutions remains open. This problem is of particular importance when we perform numerical simulations of (1) i.e., we must necessarily solve the boundary problem.

Henceforth, we shall confine ourselves to the discussion of three versions of the boundary conditions:

1) zero, when $\psi(0) = \psi(l) = 0$;
2) periodic;
3) $\psi(0) = a_0$, $\psi(l) = a_1$, where a_0 and a_1 are time-independent constants.

From the point of view of applications, the zero boundary conditions are typical for hydrodynamics. For convection this means that the edge vortex near the wall has a zero amplitude. For the Couette flow, however, the boundary conditions for ψ should be equal to a certain constant determined by the frequency of the rotation of the external cylinder than equal to zero. Near the edges of the interval for the first and third boundary conditions, the stationary solution realizing a minimum of F will significantly differ from $\psi = 1$. If the boundary conditions are periodic, then, obviously, the minimum of F is realized at $\psi = 1$. In this case among the solutions of (4) only those hold up for which $k_n^2 = (2\pi n/l)^2$ with integer n. At $k_n^2 > 1/3$ these solutions will be unstable.

In this paper we study the dynamics of defects of vortex lattices which represent transition domains between i) stable distributions of (4) and ii) unstable solutions of (4). We investigate the development of both the nonlinear stage of instability of solutions (4) and the influence of the boundary conditions on it. We show that the dynamics of such defects essentially depends on topological characteristics of the field $\psi(x)$. In the case when a defect is a region of transition between stable states, this defect expands diffusively. In contrast to such defects, a defect between a stable ($\psi = 1$) and unstable states propagates with a certain velocity oscillating in time. The mean velocity and the frequency of oscillations are analytically determined and the comparison is made with the data of numerical experiments. It is important to emphasize that the problem of the propagation by its formulation represents a generalization of the problem posed and solved by Kolmogorov, Petrovsky and Piskunov [5] for an equation of the form (1) for a real field $\psi(x)$. The method which we have used to solve this problem is borrowed by us from the work by Kamensky and Manakov [6]. It should be noted that later and independently this method was recovered in the paper[7].

2. STATIONARY STATES AND LINEAR STABILITY

Let us study the problem of stationary states and their stability.

First, let us give a brief solution of the problem of the stability of stationary states (4) assuming the interval to be infinite. Represent ψ as

$$\psi = (\tilde{\psi}_0 + \chi' + i\chi'')e^{ikx}, \tag{6}$$

where $\tilde{\psi}_0 = (1 - k^2)^{1/2}$, χ' and χ'' are the real and imaginary part of the perturbation and $|\chi'|, |\chi''| \ll |\tilde{\psi}_0|$. Then in the linear approximation we have

$$\begin{aligned}
\chi'_t &= \chi'_{xx} - 2\tilde{\psi}_0^2\chi' - 2k\chi''_x, \\
\chi''_t &= \chi''_{xx} + 2k\chi'_x.
\end{aligned} \tag{7}$$

Assuming that $\chi', \chi'' \sim e^{\Gamma t + i\sigma x}$, for the growth rate Γ we get

$$\Gamma_{1,2} = -\sigma^2 - \tilde{\psi}_0^2 \pm (\tilde{\psi}_0^4 + 4k^2\sigma^2)^{1/2}. \tag{8}$$

Hence, in particular, it follows that instability ($\Gamma > 0$) takes place [3] at

$$k^2 > 1/3, \tag{9}$$

and that stability occurs in the opposite case. In the case of a finite interval L and periodic boundary conditions the criterion (9) remains if we put $k = 2\pi n/L$, where n is an integer.

Now consider the case of zero boundary conditions, assuming that the length of the interval is $L \gg 1$. Then far from the boundary the minimum of F will be realized by a function close to $\psi = 1$ with an exponential accuracy as will be shown below. Thus, one needs to find a stationary solution which tends to $\psi = 1$ far from $x = 0$, and turns into zero at $x = 0$. It is easy to see that this solution is

$$\psi_0(x) = \tanh(x/\sqrt{2}). \tag{10}$$

Now let us demonstrate that this solution is stable in the class of functions with $\psi(0) = 0$ and $\psi \to 1$ as $x \to \infty$. In the linear approximation, $\psi = \psi_0 + \xi$, for perturbations $\xi = \xi' + i\xi'' \sim e^{-Et}$ the following spectral problem arises

$$E\xi' = -\xi'_{x'x'}/2 - (1 - 3\psi_0^2)\xi', \tag{11}$$

$$E\xi'' = -\xi''_{x'x'}/2 - (1 - \psi_0^2)\xi'', \tag{12}$$

where $x' = x/\sqrt{2}$. The first equation, Schroedinger equation, has a stable solution in the form of the shift mode $\xi'_0 = \partial\psi_0/\partial x = 1/(\sqrt{2}\cosh^2 x')$, corresponding to the ground state $E = 0$. All other eigenfunctions of (11) have $E > 0$ and, conse-

quently, are stable. The second equation of the system (11)-(12) has one bound state $\xi' = \cosh^{-1} x'$, with $E = -1/2$ and a continuous spectrum with $E > 0$. In the absence of the boundary these perturbations grow as $e^{t/2}$. However, in the presence of the boundary when $\psi(0) = 0$, symmetric solutions do not survive; there are only anti-symmetric solutions with $E > 0$, and perturbations prove to be stable. This confirms the linear stability of this solution.

It is easy to show that this solution realizes the minimum of the functional

$$F = \int_0^\infty |\psi_x|^2 + \frac{1}{2}(|\psi|^2 - 1)^2 \, dx , \tag{13}$$

on the class of functions with $\psi(0) = 0$ and $\psi(\infty) = 1$, where at F in comparison with (2) the constant term corresponding to $\psi = 1$ is subtracted.

To prove this statement it is sufficient to examine all stationary points of the functional (13) and select those that obey the necessary boundary conditions. The stationary equation associated with (1) is

$$\psi_{xx} + \psi - |\psi|^2\psi = 0 ;$$

it is easy to check that it has only one solution satisfying the necessary boundary conditions, i.e., $\psi_0 = \tanh(x/\sqrt{2})$. Hence, it follows that $\psi_0(x)$ realizes the absolute minimum of F (13) and consequently is stable according to the Lyapunov theorem.

In the case when the interval size l is large ($l \gg 1$) and the boundary conditions are zero, the solution $\psi_0(x)$ will be very close to $\tanh(x/\sqrt{2})$ in the vicinity of $x = 0$ and to $\tanh((l - x)/\sqrt{2})$ near $x = l$. In the middle of the region the solution will approximate $\psi = 1$ with an exponential accuracy. In Fig.1 we see the dependence of $|\psi|$ on x for $l = 25$ of the stationary state which arises as a result of the development of instability of small initial data. Towards the middle of the interval, the difference of ψ from 1 amounts to 10^{-6} whereas on the edges $\psi(x)$ is described with good accuracy by the dependence $\tanh(x/\sqrt{2})$.

Let the initial condition $\psi_0(x)$ represent a defect for the infinite interval. As $x \to \infty$, $\psi_0(x)$ approaches the absolutely stable solution $\psi = 1$, and at the other infinity tends to one of the solutions of (4). For vortices this defect is a region of transition between a chain of vortices having an optimal size corresponding to $k = k_0$ and a system of vortices compressed or stretched in comparison with the optimal size of the vortex. Since the state (4) has a larger value of the free energy density $\mathcal{F}$ than $\psi = 1$ such a defect will propagate in accordance with (3) towards a decrease of F, i.e., in the given case to the right. It is clear that in a finite but sufficiently large system the influence of the boundaries will not effect the defect if its size Δl is small in comparison with l. However, we should stress that over a long period of time the influence of the boundaries will become significant. The most important factor determining the dynamics of the defect is connected with topological restrictions.

3. TOPOLOGICAL CONSTRAINTS

Let us assume the boundary conditions are periodic; then the difference of phases on the boundary of the interval $\Phi = \phi(l) - \phi(0) = \int_0^l \partial\phi/\partial x \, dx$ of the field $\psi(x) = Ae^{i\phi}$ (A is the amplitude and ϕ is the phase) must be equal to $2\pi N$ for some integer N. In other words, this means that the edge of the two-dimensional vector $\mathbf{A} = (\psi', \psi'')$, where ψ' and ψ'' are the real and imaginary parts of ψ respectively, while the "motion" along the axis x from $x = 0$ to $x = l$ describes some helical line, performing N rotations around the x axis. In the case when the amplitude does not vanish anywhere, N coincides with a degree of the mapping $R^1 \to S^1$. If as a result of the evolution in t the vector $\mathbf{A}$ does not turn into zero at any point, then N is a certain integral of motion. However, if in a certain moment of time t_0 the vector $\mathbf{A}$ does turn into zero in a certain point x_0, then in this case Φ will change by 2π. From the geometrical point of view, this corresponds to the intersection of the curve described by the edge of the vector $\mathbf{A}$ and x-axis. As a result of such an intersection, the number of rotations N will change by one unit. This consideration shows that this effect does not depend on the type of boundary conditions. In any case, such a topological property, if the field $\psi(x)$ exists, its change will occur in the same manner.

Let us now consider what the solutions of (4) are from this geometrical point of view. For these solutions the vector $\mathbf{A}$ describes a helix with a constant step $h = 2\pi/k$. On the other hand, as has already been pointed out, the solutions of (4) have a smaller value of free energy than $\psi = 1$ corresponding in the three-dimensional space (ψ', ψ'', x) to a straight parallel to x-axis. From the point of view of energy, it is preferable for the solutions (4) to transit into the state $\psi = 1$. This transition must be accompanied by a phase jump by the integer N in units of 2π and by vanishing of ψ in certain moments of time t_{0i} at a certain point x_{0i}. This transition is possible if a strong instability exists since the state $\psi = 0$ is unstable. We should remember that for the solution $\psi_0 = \tanh(x/\sqrt{2})$ the point $x = 0$ occurs as a saddle: along one direction (real) there is attraction; along the other (imaginary) we have repulsion. In Figs.2-4 are the results of simulations of Eq. (1) with periodic boundary conditions for the initial data $\psi = (\tilde{\psi}_{0k} + \epsilon \cos x)e^{ikx}$ with $k = 14 * 2\pi/125$ from the unstable region, $\sigma = 10 * 2\pi/125$ and $\epsilon = 0.02$. Fig.2 shows the dependence of $|\psi|$ on x for three moments of time when $|\psi|$ becomes zero. Fig.3 shows the time dependence of $U = min_x|\psi|$. In the moment of time when u touches the x-axis, the phase Φ changes to 2π by a jump. The phase reduction occurs sufficiently quickly, reaching a certain stable N_{st} corresponding to $k_{st} = 2\pi N_{st}/l$. For this run $N_{st} = 2$. After this process we can observe a slower diffusive relaxation tending to the state $\psi = \psi(k_{st})$.

4. DEFECT PROPAGATION

From the aforementioned it becomes clear that the propagation of the defects with some velocity is possible only if one of the states is unstable. If both states
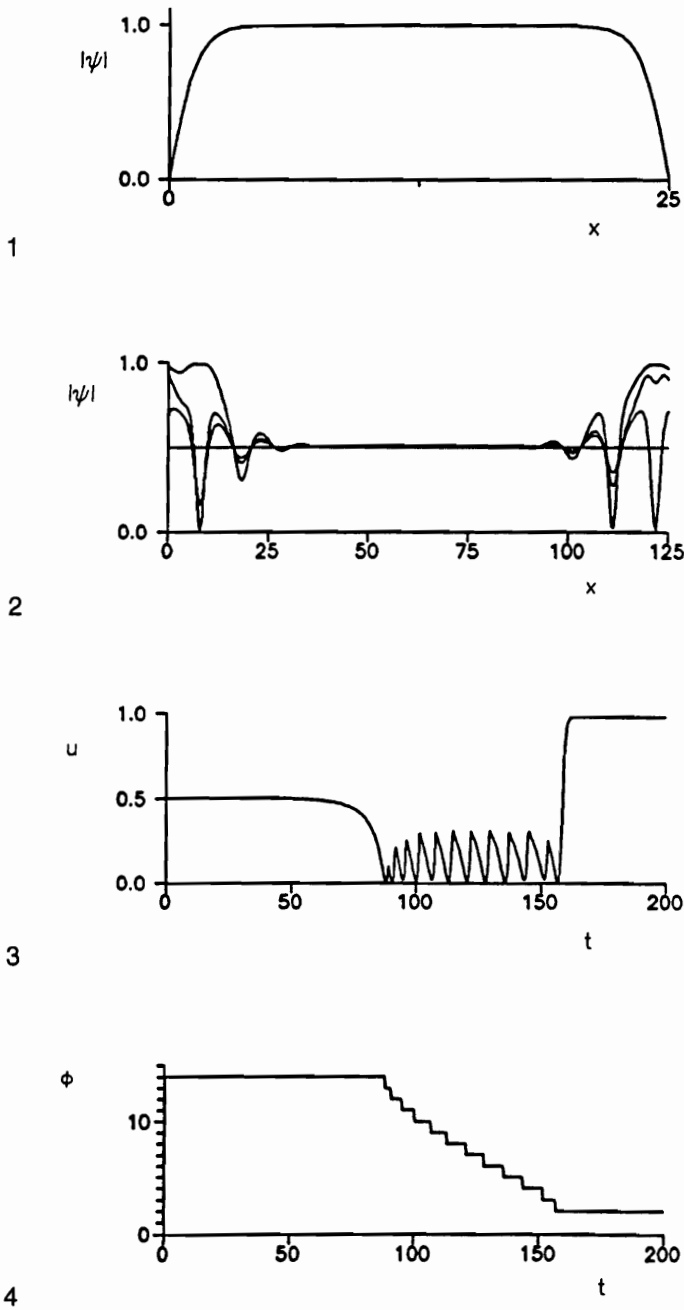
Fig.1-4. Dependencies of $|\psi|$ in successive moments of time, dependencies of $U = min_x|\psi|$ and phase Φ in 2π units as functions of time for the instability development of the solution $\psi_k = (1 - k^2)^{1/2}e^{ikx}$ with $k = 14 * 2\pi/125$. The vanishing of U and the jumps in phase Φ of 2π take place at the same moments of time.

are stable, there is no reason for the defect to move. Its dynamics will be essentially different from the first case.

Let the initial conditions be such that $\psi_0 \to 1$ as $x \to -\infty$ and $\psi_0 \to \psi_k = (1 - k^2)^{1/2} e^{ikx}$ as $x \to \infty$, $|k| > 1/\sqrt{3}$. Then the interface will propagate to the right with a certain velocity. To find the velocity we shall assume that far from the front in the unstable phase the initial conditions will differ only slightly from ψ_k. Slightly means that $\Lambda = \ln(|\psi_k/\delta\psi|) \gg 1$ where $\delta\psi$ is the perturbation. This assumption permits us to consider that the linear stage of the instability (8) lasts long enough and can be described by means of the saddle point method. Knowing the solution before the front it is necessary to match it to the main wave coming onto the asymptotics $\psi \to 1$ as $x \to -\infty$. This problem is analogous by its formulation to the Kolmogorov-Petrovsky-Piskunov problem [5] but has a principal difference. As is shown by the numerical experiment, the defect moves, on average, with a constant velocity V. In Figs.5-9 are the data for version $k = 0.95$. At the motion of the defect in its front, we can observe periodic variations with a frequency which can be estimated as $\omega_D = kV$. In this situation, after each period of oscillations there occurs a phase-slip of 2π, i.e., the spiral uncoils (Figs.5,6). Thus on the front of the wave there are complicated nonlinear oscillations. Apart from them in the value of the velocity we can also observe certain oscillations, but with a smaller frequency. After the main front is gone, the state after the front is different from $\psi = 1$, and there is still a certain residual rotation (Figs.8,9). To define the value of the mean velocity of the defect let us use the method proposed in [6]. For this purpose, consider a solution far from the front in the region of instability, where perturbations are small: $\Lambda = \ln|\psi_0/\delta\psi| \gg 1$. To find the velocity, let us require that in the reference system, moving with the velocity of the defect V, perturbations do not grow exponentially in time. In this instance, we shall select the solutions that will grow exponentially over x while approaching the main front of the defect. It is principally important that to find the velocity of the defect it is not necessary to solve the problem of matching with the main wave; the velocity is obtained from the analysis of only the linear problem.

So, transforming to the reference system moving with the velocity V, from Eqs.(7)-(8) for the perturbations χ', χ'' we get

$$
\begin{pmatrix} \chi' \\ \chi'' \end{pmatrix} = \int_{-\infty}^{\infty} \left[\left(\frac{\Gamma_1 + \sigma^2}{2ik\sigma} \right) C_1(\sigma) \exp\left(\Gamma_1(\sigma)t + i\sigma x' + i\sigma Vt \right) \right.
$$
$$
\left. + \left(\frac{\Gamma_2 + \sigma^2}{2ik\sigma} \right) C_2(\sigma) \exp\left(\Gamma_2(\sigma)t + i\sigma x' + i\sigma Vt \right) \right] d\sigma .
$$
(14)

Here $x' = x - Vt$, and the functions $C_1(\sigma)$ and $C_2(\sigma)$ are defined from the initial conditions.

Since the initial noise is small ($\Lambda \gg 1$), the linear stage of the instability lasts a long time $\Lambda \Gamma_{\max}^{-1}$. Therefore, far from the front we can confine ourselves to the linear
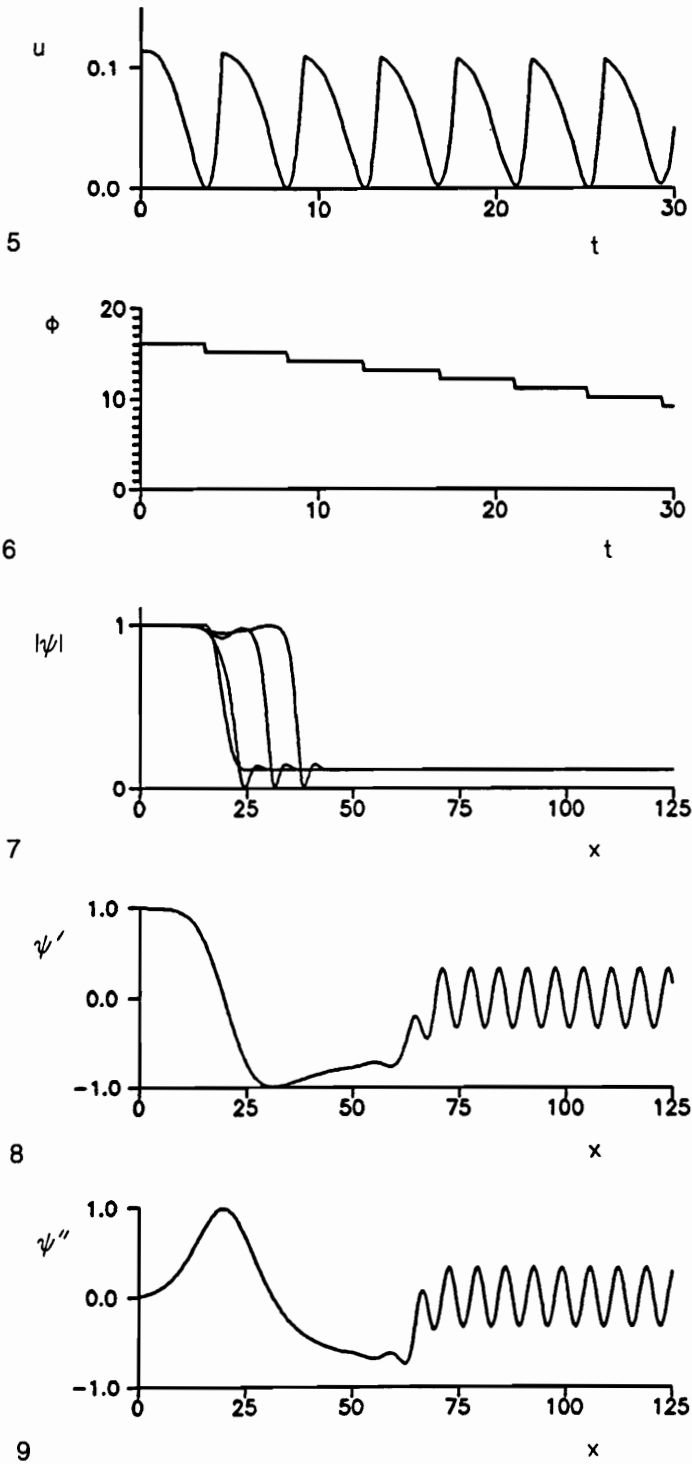
Fig.5-9. Dependencies of U, Φ, $|\psi|$, ψ' and ψ'' for the propagation of the defect with $k = 0.95$. Boundary conditions: $\psi(0) = 1$, $\psi(l) = \psi_k(l)$.

approximation and find the solution in terms of the integral (14) as $t \to \infty$ by using the saddle point method. In this case, it is necessary to make the cut through the points $\sigma = \pm i\psi_0^2/2k$ and to glue the edges of the cut. After this, we can use the saddle point method.

The saddle point is found from the condition

$$\Gamma'(\sigma) + iV = 0. \tag{15}$$

This equation reduces to the equation of the fourth power with respect to σ. Among the roots of this equation we should choose a σ_m for which the value of $Re(\Gamma(\sigma) + i\sigma V)$ could be maximal.

This saddle point gives the maximal contribution to the integral (14). As a result, the perturbation with the accuracy up to the preexponential factor (irrelevant for our further investigation) behaves as

$$\exp\left\{ Re(\Gamma(\sigma_m) + i\sigma_m V)t + i\omega_m x' + ik(x' + Vt) \right\}, \tag{16}$$

$$\omega_m = Im(\Gamma(\sigma_m) + i\sigma_m V).$$

We have included in this expression the exponential factor connected with the change (6).

The absence of the exponential growth over t in (16) determines the value of the velocity of the wave:

$$Re(\Gamma(\sigma_m) + i\sigma_m V) = 0. \tag{17}$$

At a velocity smaller than V, defined from (17), perturbations will grow exponentially and, consequently, the process of the propagation will not be of a quasi-stationary character. Given a large V, perturbations will not have time to develop into a wave. Therefore, the requirement (17) defines the value of the velocity of the wave. It should be noted that $Im\sigma_m > 0$. This means that, at the approach to the front of the defect, the solution grows exponentially as a function of x. From (16) it is also evident that perturbations, apart from the Doppler frequency kV, have the frequency ω_m.

Eqs.(15),(17) can be studied analytically in two limits:

$$1 - k^2 = |\psi_k|^2 \ll 1,$$

and

$$k^2 - 1/3 = \epsilon \ll 1.$$

In the first limit $\Gamma(\sigma)$ can be written approximately in the form

$$\Gamma(\sigma) \approx -\sigma^2 - 2k\sigma - \psi_k^2.$$

Substituting this expression in (15), we find the quantity

$$\sigma_m = -iV/2 - k.$$

The insertion of σ_m into (17) for the velocity V yields

$$V = 2(1 - |\psi_k|^2).$$

Then the Doppler frequency equals

$$\omega = 2k(1 - |\psi_k|^2).$$

To calculate the frequency ω_m, it is necessary to retain the following corrections over $|\psi_k|^2$ in the expression for $\Gamma(\sigma)$. Simple calculations yield

$$\omega_m = |\psi_k|^4/4.$$

In the other limit the growth rate can be replaced by

$$\Gamma(\sigma) = \frac{9}{2}\epsilon\sigma^2 - \frac{3}{16}\sigma^4.$$

In this case it is convenient in the expression

$$\Gamma(\sigma) + i\sigma V = f(\sigma),$$

to introduce new variables θ and y: $\sigma = (12\epsilon)^{1/2}y$, $V = \theta\epsilon^{3/2}9\sqrt{12}$. As a result,

$$f(\sigma) = 108\epsilon^2 g(y),$$

$$g(y) = y^2/2 - y^4/4 + i\theta y.$$

After simple transformations, Eqs.(15,(17) can be solved as

$$y = \left((\sqrt{7}+3)/4\right)^{1/2} + i\left((\sqrt{7}-1)/12\right)^{1/2},$$
$$\theta = (\sqrt{7}+2)(\sqrt{7}-1/3)^{1/2}/3.$$

The results of the numerical calculation of Eqs.(15),(17) are given in Figs.10-12. In Fig.10 the solid line corresponds to the values of the velocity V, calculated from (15), (17). The asterisks mark the values of the velocity measured in numerical experiments. The mean velocity has been defined as the ratio of the distance propagated by a defect per a period to the period of oscillations. This period of oscillations has been measured as the time between two successive phase jumps (see Fig.6). In Fig.11 the solid line stands for the Doppler frequency, calculated by means of Eqs.(15),(17). The asterisks mark the frequency determined by the period of oscillations of the quantity $u = \min_x |\psi(x,t)|$. In both graphs for V and ω_D, there is good agreement between theory and numerical experiments. Finally, Fig.12 shows the dependence of the frequency ω_m on the wave number k. In numerical experiments we have observed a frequency close to ω_m, which corresponds to oscillations of the velocity V with respect to the mean value. These oscillations amount to 5% and have a tendency to decay.
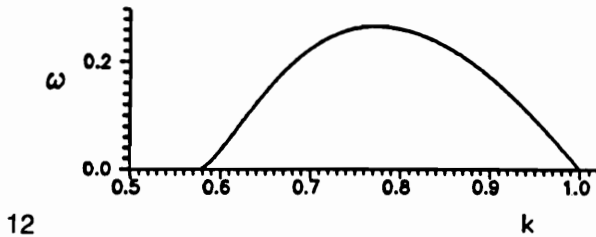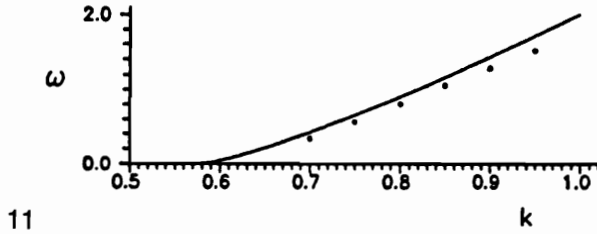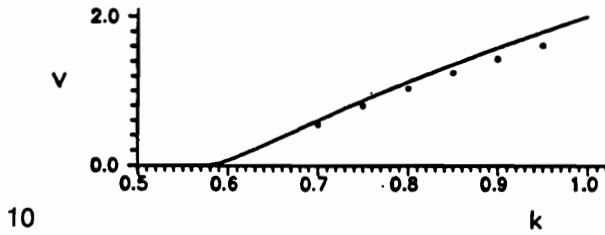
Fig.10–12.

Now let us briefly study what effects occur for the defects between two stable vortex lattices. The principal distinction of the defects studied above consists of the absence of a strong instability. Therefore the defect cannot propagate by virtue of the topological constraints. In this case, there occurs a slow diffusive unwinding of the helix. Figs.13-15 give the spatial distributions for $|\psi|$, the real part ψ', and the imaginary part ψ''. On the edges of the interval $\psi(x)$ was constant: $\psi|_{x=0} = 1$, $\psi|_{x=l} = \psi_k(l)$. The transition region was expanding in time, which corresponded to the unwinding of the helix. In the region where ψ was equal to one, there was rotation, which is absolutely clear from Fig.14-15. The real part in this region became smaller than one, and the imaginary part, on the contrary, grew. These results are in full agreement with the conclusion of the paper [8] — such defects expand diffusively in time.
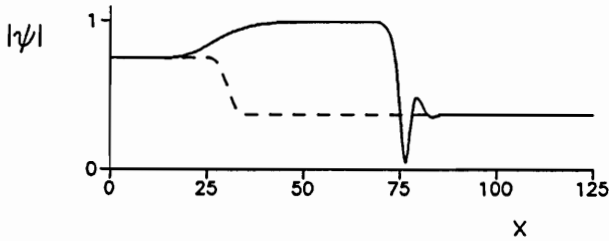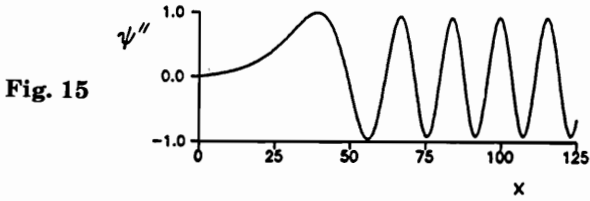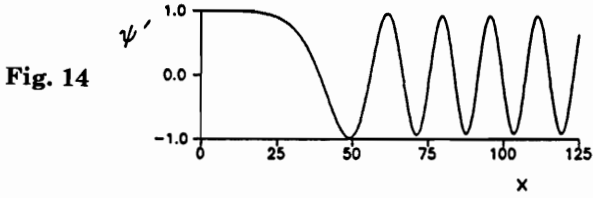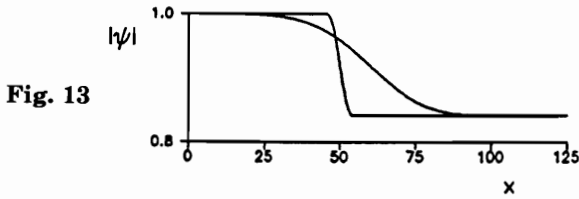
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Fig. 13

Fig. 14

Fig. 15

Fig. 16. Distributions of $|\psi|$ for the decay of the initial condition (dotted line) in the form of defect with $k_1 = 0.5$ and $k_2 = 0.8$. Boundary conditions: $\psi(0) = \psi_{k_1}(0)$, $\psi(l) = \psi_{k_2}(l)$.

Let us consider now how the defect with arbitrary values k_1 and k_2 will decay if k_1 lies in the unstable region but if k_2 belongs to the stable one. As numerical experiments showed (see Fig.16), the defect begins to propagate into the unstable region with the parameters defined with the help of formulae (16),(17). The state behind the defect front has the amplitude close to $|\psi| = 1$ with some residual rotation. Between this state and the wave defined by k_1 from the stable region, the defect of the first type forms with a diffusive spreading front. The decay of such initial conditions leads, thus, to the formation of two types of defects. The parameters of the defects can be defined independently.

5. CONCLUSION

We have clarified that the dynamical properties of the defects for vortex lattices depend on whether the topological invariant of the complex field $\psi(x)$, coinciding up to the constant multiplier 2π with the integral phase difference of $\psi(x)$ on the interval edges, conserves or does not conserve. The reason for defect propagation is just connected with nonconservation of the invariant, i.e., its reduction. If the topological invariant conserves then the defect will spread diffusively.

It should be noted also the considered in this paper topological effects are intrinsic for two-dimensional (1+1) models for a complex field. For such cases the transition mechanism from we state with the invariant N_1 to the other with N_2 is common — the phase of this field ψ has to change by 2π with each touching of the x-axis by $|\psi|$. It should be emphasized that such a transition is only possible for the strong instability existence. The nonlinear Schroedinger equation with attraction and repulsion and its different generalizations represent the examples of such systems.

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