Memristor – New Electronic Device

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Classical Circuits

Four Fundamental Circuit Variables:
1. Charge – q
2. Voltage – v
3. Current – I
4. Magnetic Flux – φ

Three Classical Fundamental Circuit Elements:
1. Capacitor: dq = C dv
2. Resistor: dv = R di
3. Inductor: dφ = L di

Two Additional Relationships:
1. dq = i dt
2. dφ = v dt

Modeling of RLC Circuit with Applied Voltage

L Q” + R Q’ + Q/C = V(t)
Existence of the Memory Resistor

\[ v = \int M(q) \, dq \]

- An element that is:
  - Passive
  - Dissipative
  - 2-terminal
- Effectively a resistor with resistance that depends on charge that has passed through - memory

Existence of the Memory Resistor

\[ v = \int M(q) \, dq \]

- Applications
  - Higher density circuits
  - Unique modeling possibilities
  - Generation of special waveforms
  - Signal processing

Memristor - The Fourth Element

• Proposed by Leon Chua in 1971 using an argument based on symmetry.
• Described by the sixth relationship between the four fundamental circuit variables: $d\phi = M dq$
• Faraday’s Law of Induction states the induced EMF or voltage in a closed circuit is equal to the time rate of change in magnetic flux. Therefore, the memristor equation can be expressed as the following:

$$v = M(q)i$$

• Similar equation to resistors described by Ohm’s Law ($v = R i$). Memristance can reduce to resistance if certain conditions are met.
• Memristor combination of “memory” and “resistor”.
• Symbol for memristor:

Properties of Memristors

- Non-linear relationship between current and voltage.
- Reduces to resistor for large frequencies as evident in the i-v characteristic curve. May also reduce to a resistor based on defined state variables.
- Memory capacities based on different resistances produced by the memristor.
- Non-volatile memory possible if the magnetic flux and charge through the memristor have a positive relationship (M > 0).
- Does not store energy.
- Similar to classical circuit elements, a system of memristors can also be described as a single memristor.

i-v Characteristic Curve

- Pinched hysteresis unique to memristors.
- No combination of other fundamental circuit elements makes Lissajous figure.

Chua, LO, Kang SM Proceedings of the IEEE 64 Issue 2 (1976)
Generalization of Memristance

Recall the derived equation for memristance:

$$v = M(q)i$$

This can be generalized further by considering a set of state variables $x = (x_1, x_2, ..., x_n)$. These state variables are dependant on the specific implementation of the memristor. We can use the state variables to make a substitution in our memristance equation.

$$v = M(x)i \quad \frac{dx}{dt} = f(x,i)$$

The state variables must be related to the current. This generalization leads to a unique set of equations for different memristors and memristive systems.

Chua, LO, Kang SM Proceedings of the IEEE 64 Issue 2 (1976)
Consider a light bulb. In general, a light bulb can be thought of as a resistor. However, as the filament heats up, the resistance of the bulb increases. This behavior creates a non-linear resistance which can be described with the following temperature-dependant equations:

\[ V = (R_0T)I = M(T)i \]

\[ \frac{dT}{dt} = aTi^2 - b(t^4 - t_0^4) = f(T,i) \]

where \( R_0, T_0, a, \) and \( b \) are constants. These equations satisfy our conditions for a memristive system.

Cunningham, W.J. Journal of Applied Physics (Vol. 23, No. 6) 1952
Difficulties in Finding Memristors

- Why has it taken almost four decades to find a memristor?
- Memristors are not new. Memristive properties have been observed by researchers for more than three decades.
- Pinched hysteresis curve is a unique property of a memristor. However, researchers did not make the connection with their observations.
- Often described as anomalous inductance/resistance and disregarded in certain practical applications (e.g. Josephson Junction).
- Direct link between charge and magnetic flux not necessary.

Formation and Applications

- Physical model
- Electroforming
- Light emitting memristor
- Memristors in logic gates
- Modeling simple learning
Memristor Found

\[ R_{\text{Total}} = R_{\text{ON}} \frac{w(t)}{D} + R_{\text{OFF}} \left(1 - \frac{w(t)}{D}\right) \]

\[ v(t) = \left(R_{\text{ON}} \frac{w(t)}{D} + R_{\text{OFF}} \left(1 - \frac{w(t)}{D}\right)\right)i(t) \]

\[ \frac{dw(t)}{dt} = \mu_v \frac{R_{\text{ON}}}{D} i(t) \]

\[ w(t) = \mu_v \frac{R_{\text{ON}}}{D} q(t) \]

\[ M(q) = R_{\text{off}} \left(1 - \frac{\mu_v R_{\text{on}}}{D^2} q(t)\right) \]

Formation of TiO$_2$ Memristor

$$\text{TiO}_2 \xrightarrow{V} \text{TiO}_2-x + \text{O}_2$$

Yang, et. al. Nanotechnology 20 (2009) 21501
Formation of TiO$_2$ Memristor

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Light Emitting Memristor

Memristive Logic Gates

Simple “Learning” Circuit

Simple “Learning” Circuit

Simple “Learning” Circuit

HP Simulations

HP Nature Paper

Our Simulation

HP Simulations Cont.

HP Nature Paper

Our Simulation

HP Simulations Cont.

HP Nature Paper

Our Simulation

Future Directions

- More precise modeling
- Learning circuits in parallel – signal processing
- Additional voltage waveforms
References

- Cunningham, W.J. Journal of Applied Physics (Vol. 23, No. 6) 1952