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Electromechanical Properties of Bones

Abstract

Bones are an integral part of the human body that exhibit piezoelectric properties, meaning that when stress is applied to the bone, the bone produces a current within itself. This is known as the piezoelectric effect. Material that are piezoelectric can similarly exhibit the reverse piezoelectric effect, meaning that when a current is applied, the material compresses on itself. This property has many applications such as applying a current to a fractured bone to promote compression of the bone to allow it to heal faster. However, too much of a current will destroy the tissue of the bone, thus killing the cells. The goal of our research is to determine the electrical conductivity of the human bone to establish the maximum amount of current that can be applied before the bone tissue begins to become damaged. Our model will expand upon that which is described by R. Casas and I. Sevostianov by considering the interaction between the Haversian and Volkmann canals of bones and their effect on the overall conductivity of the bone.

Introduction

It is well known by everyone that bones are a crucial component to the human body. Bones provide stability of structure so that humans may stand upright and bones also provide protection for vital organs such as the brain, lungs and heart. Red and white blood cells are also made within bone marrow so bones can be seen to house important physiological processes as well. Though the importance of bones is well known, intensive study on bone structure, also known as osteology, only began within the last century. These studies led to the discovery of many intriguing properties which commands further exploration.

One of the properties discovered, was that bones are dielectric. This implies that they themselves are semi-conductive. In addition, they are anisotropic, so not only are bones conductive, they conduct differently depending on the direction of the flow of the current through the bone. The major find was that bones are piezoelectric materials. This piezoelectric property implies that a voltage could be applied to the bone and the bone itself would compress. It is also well known that if a bone is fractured, compressing the fracture shortens the healing time. So in theory, if there were many small fractures in the bone, a voltage could be applied to the bones which would then cause the bone to compress and allow the fractures to heal quicker. However, since bones are semi-conductive this means that if a voltage is applied to a bone, a current will be produced which could cause damage to tissue inside the bone which would effectively kill the bone and prevent it from healing. Therefore, the motivation behind the model is to accurately describe the flow of any current in the bone so that the exploitation of the piezoelectric property can be safely applied as a treatment.

Before modeling the flow of current, certain physical properties of the bone needs to be addressed. The structure of bone is quite complicated and can be seen as a network of different parts all intertwined in a specific manner. Since the goal is to model the total conductivity of a bone, the structure of the bone can be simplified as a sum of three different parts: Haversian canals, Volkmann canals, and the rest of the bone which will be referred to as the bone matrix. The Haversian canals run parallel to the length of the bone, are an average 5 mm in length, and are approximately 50 microns in diameter. Volkmann canals connect Haversian canals together and they are orientated in the perpendicular direction of the Haversian canals. Volkmann canals are about 0.5 mm in length with an average diameter of 5 microns. The aspect ratio, the ratio of length to diameter, of the Haversian and Volkmann canals are both equal to 100 and, they have the same filling factor of 2%. Since the canals contain capillaries and nerve tissues which both are much more conductive than the bone matrix, the canals can be approximated to be metallic. The provided information allows the start of the formation of an accurate model.

Before current can be modeled the bone must first be simplified even further so that electrodynamic techniques can be properly used. Since Casas and Sevostianovs' paper, **Electrical resistivity of cortical bone: Micromechanical modeling and experimental verification,** was used as a reference when constructing the model, their bone model as well as an alternative bone model will be described along with the benefits and pitfalls of each model. In Casas and Sevostianovs' paper they made three assumptions; one assumption was that the total conductivity of the bone matrix was σ, independent of direction, another assumption was that no canals had any interactions with each other and therefore no canals intersected and the last assumption, was that each canal was viewed as an elongated ellipsoid. Casas and Sevostianov then, using well know electrodynamic equations, calculated the contribution of conductivity of one Haversian canal, using the fill factor and averaging techniques, calculated the average contributions to conductivity of all the Haversian canals. They used a similar technique and found the contribution to conductivity of all the Volkmann canals and then summed the three contributions (bone matrix, Haversian canals, and Volkmann canals) together to equate the total conductivity.

The model becomes simple and straightforward so finding parameters becomes almost trivial which makes the model as one of the most attractive choices however there is a flaw in the assumptions. One of the assumptions was the the canals have no interactions with one another however if there are no intersections of canals, specifically Volkmann and Haversian canals, then biologically the bone in study would be dead and no longer heal. This implies that any answer derived from the model may have substantial error in it. Therefore, in order for the model to be accurate it must account for at least some interaction between the Volkmann and Haversian canals.

An alternative view of the bone is conductive ellipsoids inside a semi-conductive matrix. In this model the canals are also viewed as elongated ellipsoids however, instead of viewing the canals as separate, the Volkmann canals will be part of the matrix, giving it a uniform conductivity perpendicular to the Haversian canals. The rationale involves using the fill factor of the canals. Since the canals have the same fill factor but differ in size by a large magnitude, this implies that there is a significantly higher quantity of Volkmann canals then there are Haversian canals. Due to this significant difference in amounts of the canals and the large difference of conductivity between canals and the bone matrix, it can be approximated that the Volkmann canal and the bone matrix can be viewed as a homogenized substance with a new conductivity, φ, that points along the axis perpendicular to the Haversian canals. For this approximation it must be assumed that the number of of Volkmann canals are roughly uniform within their plane. This assumption isn't drastically far fetched since there is no biological reasoning against it. After the homogenization, the bone can be viewed as a set of conductive ellipsoids inside a lesser conductive matrix whose direction of conductivity is perpendicular to the ellipsoids. This model better accounts for the interaction between the two canals and is still relatively simple since the electrodynamic equations for the system are well known.

However no matter which bone model is used there is still the issue of dealing with charge flowing in different directions due to the anisotropic nature of the bone. Unlike a metallic rod where the current will flow in a unidirectional fashion, when current travels along a bone it will branch out and go in different directions making it more complicated to track all of the current and every flow. Another technique must be introduced so that the problem can become manageable.

Explanation of Tensors

As previously discussed, bones are anisotropic, meaning that the conductivity of the bone may differ dependent on the direction considered. This creates a problem in modeling the electrical conductivity of the bones. To deal with this problem, the mathematical concept of tensors is introduced. Tensors, in a basic definition, are multidimensional arrays that describe some physical property. In our case, tensors will help us to describe the electrical conductivity of the bone in any direction. The rank or order of the tensor describes the dimensions of the array itself and is dependent upon the number of indices needed to describe the array. In general, a rank zero tensor is a scalar, a rank one tensor is a vector, and a rank two tensor is a matrix. One important feature of a tensor is that when using a tensor in a mathematical expression, the result should remain the same no matter the space that the tensor is used in.

There are three types of tensors that describe this, contravariant, covariant, and mixed tensors. Contravariant tensors describe the change in a displacement vector from one coordinate space to another. For example;

$$A'^{\mu} = \frac{\partial x_{\mu}}{\partial x_{\sigma}} \cdot A^{\sigma}$$

$$\binom{\mu = 1,2}{\sigma = 1,2}$$

Describes a transformation from the coordinate system σ to the μ coordinate system by describing the change of the two coordinate basis vectors in μ with respect to the two coordinate basis vectors in σ. In this example, this describes a rank one contravariant tensor, which is denoted by the index labeled as a superscript. This definition uses the Einstein summation convention, which states that anytime a product between two objects contains the same index as a subscript on one term and a superscript on another, (or in this case in the numerator of one term and the denominator of another) it is implied that the product is summed over all possible values of the index. For example;

$$y = \sum_{i=1}^{3} c_i x^i = c_1 x^1 + c_2 x^2 + c_3 x^3$$

Similarly, a covariant tensor describes the transformation of a gradient vector from one coordinate space to another and is described mathematically as;

$$A_i' = \frac{\partial x_j}{\partial x_i'} A_j$$

This describes a rank one covariant tensor, or covector, that describes the change of a gradient vector in coordinate space j to the coordinate space i. A covariant tensor is described by the index placed as a subscript. A mixed tensor is a tensor that has indices as both a subscript and superscript. Therefore, the smallest rank that a mixed tensor can have is a rank of two. The usefulness of tensors can be seen in this one property in that we can describe some physical property in a coordinate space that is more intuitive and then transform it to a space that has more physical meaning. For example, when describing the flux of water through a cylinder, a good approach may be to describe the flux out of the cylinder in cylindrical coordinates where computation will be simpler and then transform the result into cartesian coordinates. The primary tensors used in the model in the paper by R. Casas and I. Sevostianov are the resistivity contribution tensor, **R** and the conductivity contribution tensor, **K**, which is further described with respect to Eshelby's tensor $\mathbf{s}^c$.

Model on the Conductivity of Bone

The model created by R. Casas and I. Sevostianov first uses Maxwell's equations to derive the dual equations used to represent the divergence of the electric field, and the electric current density. Because the bone is dielectric, the volume of the bone is inversely proportional to the electric field, and is also inversely proportional to the electric field gradient. However, one must account for the highly conductive Haversian and Volkmann canals whose volumes are proportional to the electric field;

$$\Delta \boldsymbol{E} = \frac{V^*}{V_0} \boldsymbol{R} \cdot \boldsymbol{J}$$

Naturally, the above considerations also apply to the equation for the electric field gradient when expressed in its alternate form;

$$\Delta \boldsymbol{J} = \frac{V^*}{V_0} \boldsymbol{K} \cdot \boldsymbol{E}$$

Due to the anisotropic nature of the bone, one cannot model the resistivity and conductivity of the bone in a unidirectional fashion. As such, one must use a resistivity contribution tensor, **R**, for the inhomogeneity, and a conductivity tensor, **K**, for the inhomogeneity (both of rank-2). Resistivity and conductivity are inverses of one another, and the conductivity tensor **K** is simply the inverse tensor of the resistivity tensor R. For ease in modelling, one can approximate the Haversian and Volkmann canals as ellipsoidal in shape. This allows one to use Eshelby's results for an inhomogeneity of ellipsoidal shape to model the conductivity of the bone, where the conductivity of the matrix material (k_0), the conductivity of the inhomogeneity (k_1), and Eshelby's tensor are all considered;

$$\boldsymbol{K} = k_0\left(\boldsymbol{s}^{\mathcal{C}} - \frac{k_0}{k_1 - k_0}\boldsymbol{I}\right)^{-1}$$

In the case of the bone, the conductivity of the Haversian and Volkmann canals is much greater than that of the bone material so k_1 in the model is much greater than k_0. For relatively large values of k_1, we can approximate by taking the limit as k_1 goes to infinity, which is as follows;

$$\boldsymbol{K} = k_0\left(\boldsymbol{s}^{\mathcal{C}}\right)^{-1}$$

Since the K-tensor is proportional to the inverse of the Eshelby tensor for an inhomogeneity, this tensor must be expressed in useful terms. In the case of a spheroidal inhomogeneity with a certain aspect ratio,γ, relevant results were provided by Carlslaw & Jaeger. **n** is the unit vector with respect to the spherical axis of symmetry (which in this case is the minor axis). f_0 represents the aforementioned spherical axis of symmetry.

$$\boldsymbol{s}^{\mathcal{C}} = f_0(\boldsymbol{I} - \boldsymbol{nn}) + (1 - 2f_0)\boldsymbol{nn}$$

f_0 is itself an expression wherein shape factor g of the spheroid is considered and taken into account;

$$f_0 = \frac{\gamma^2(1-g)}{2(\gamma^2 - 1)}$$

When the aspect ratio is > 1, a prolate spheroid is being described, and when it is < 1 it is an oblate spheroid. These two cases are considered below;

$$g = \begin{cases} \frac{1}{\gamma\sqrt{1-\gamma^2}}\arctan\frac{\sqrt{1-\gamma^2}}{\gamma}, & \text{oblate shape } (\gamma < 1) \\ \frac{1}{\gamma\sqrt{\gamma^2-1}}\ln\left(\gamma + \sqrt{\gamma^2 - 1}\right), & \text{prolate shape } (\gamma > 1) \end{cases}$$

As the paper approximates the canals to be prolate spheroids with a very large aspect ratio >>> 1, one can approximate the value of g by taking the limit as γgoes to infinity. Since the growth in the denominator expression greatly outstrips the growth in the numerator, this term goes to zero. Using this value of g, one can then approximate f_0 by plugging in the g that is acquired, and then taking the limit as γ goes to infinity. This value, in turn, can then be plugged into the Eshelby tensor, resulting in the following expression;

$$\boldsymbol{s}^{\mathcal{C}} = f_0(\boldsymbol{I} - \boldsymbol{nn})$$

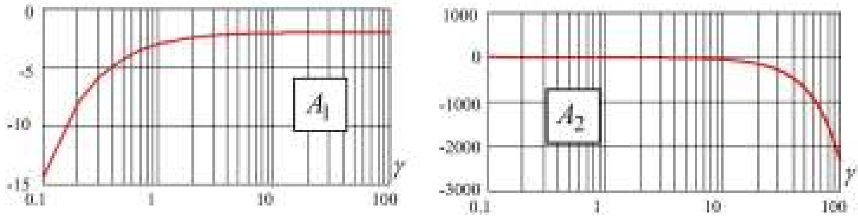
Applying dimensionless analysis, the paper arrives at the following expression of the K-tensor with dimensionless factors A_1 and A_2 where A_1 is proportional to the conductivity of the matrix material and A_2 is proportional to the conductivity of the inhomogeneity:

$$\boldsymbol{K} = -k_0(A_1\boldsymbol{I} + A_2\boldsymbol{nn})$$

$$A_1 = \frac{k_0 - k_1}{k_0 + (k_1 - k_0)f_0}, \quad A_2 = \frac{(k_0 - k_1)^2(1 - 3f_0)}{[k_1 - 2(k_1 - k_0)f_0][k_0 + (k_1 - k_0)f_0]}$$

Since A_1 and A_2 are expressed in terms of f_0, k_0, and k_1, they can be reduced further as $k_0 <<< k_1$. The behavior of A_1 and A_2 given an aspect ratio is shown in the graph below:

$$A_1 = \frac{-1}{f_0}, \quad A_2 = \frac{1-3f_0}{f_0(1-2f_0)}$$



Future Work

The model presented by R. Casas and I. Sevostianov gives a clear model on the overall conductivity of a cortical bone. However, as previously stated, one of the biggest assumptions made in this model is that there is no interaction between both the Haversian and the Volkmann canals. This assumption allows for simplification in the creation of the model but then dismisses the idea that the bone therefore cannot receive nutrients that would be transferred between the canals, rendering the bone inactive. The model that we will present will consider the interaction between the two canals in addition to the effect that the Haversian canals will have on the electric field applied to the bone.

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