Stability of Assorted Driven Pendulums

Ramon Driesen, Jaden Johnston, Massimo Pascale, Evan Ridley

Project Description

- Pyotr Kaptiza was the first to solve for the stability of the upright fixed point of a pendulum oscillating vertically in 1951. [1]
- Has been explored various other oscillations, including at arbitrary angles.
- The objective is to reproduce the pendulum oscillations at arbitrary angles experimentally. [2]
- In addition, other extensions of Kaptiza's pendulum are explored, specifically the charged pendulum.

Scientific Challenges

- The driven pendulum was the pioneer of fast and slow mechanics.
- This means that the driven pendulum (with charges) can work as an analog for parts of particle physics and other fast and slow attractors
- The implementation of a charge potential to the system is the first step towards creating other physical analogs, such as the ion trap.

Potential Applications

- Correction of instrumental angle
- Physical analog of an ion trap
- Any driven oscillations in the field of an inverse-square law

Mathematical Model

- We construct equations of motion and effective potential as described in Methodology.
- Minima of the effect potential are used to find stability angles, as shown in plots below.
- Equation of Motion: $\ddot{\theta} + \frac{3}{2}\frac{g}{l}\sin(\theta) + \frac{3}{2}\frac{d\omega^2}{l}\cos(\omega t)\,\sin(\theta-\phi) = 0$
- Effective Potential: $U_{eff} = -\frac{1}{2}mgl\cos(\sigma) - \frac{3}{32}md^2\omega^2\cos(2\sigma-2\phi)$

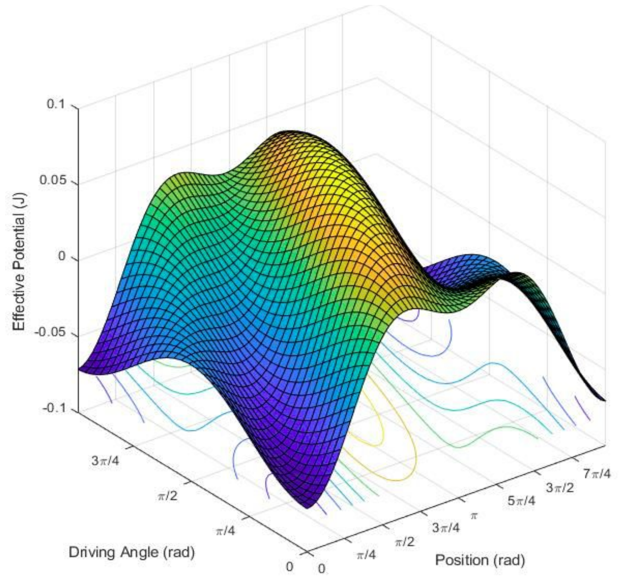

Fig 1: Map of Potentials for the arbitrary drive-angle pendulum

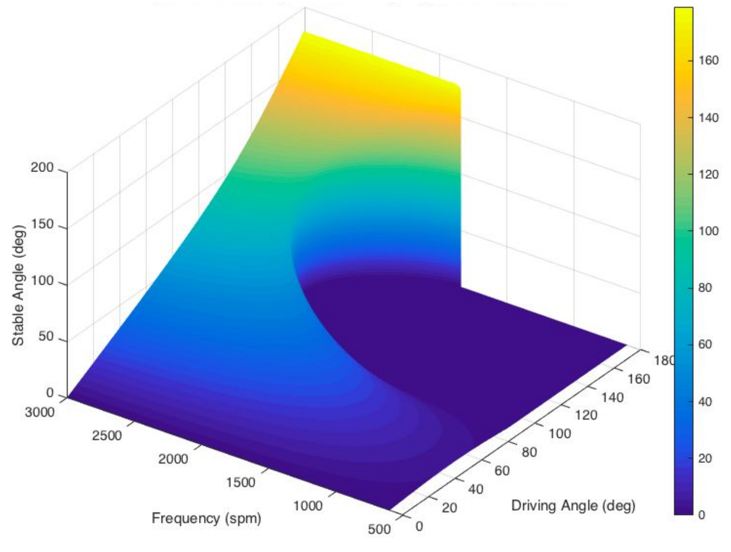

Fig 2: Stable angles for the arbitrary drive-angle pendulum

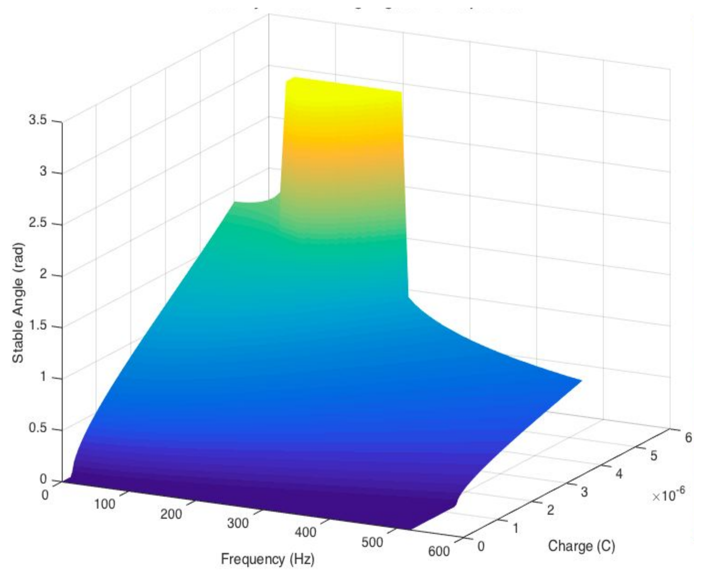

Fig 3: Stable angles for the charged pendulum

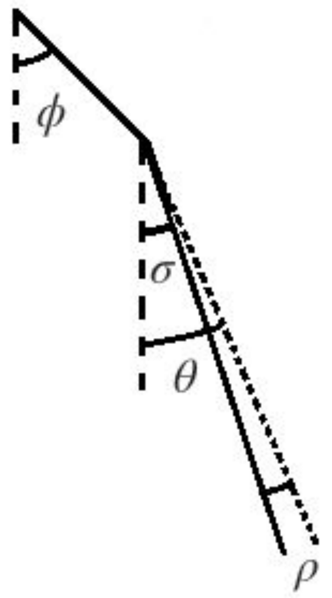

Fig 4: Averaging method set-up.

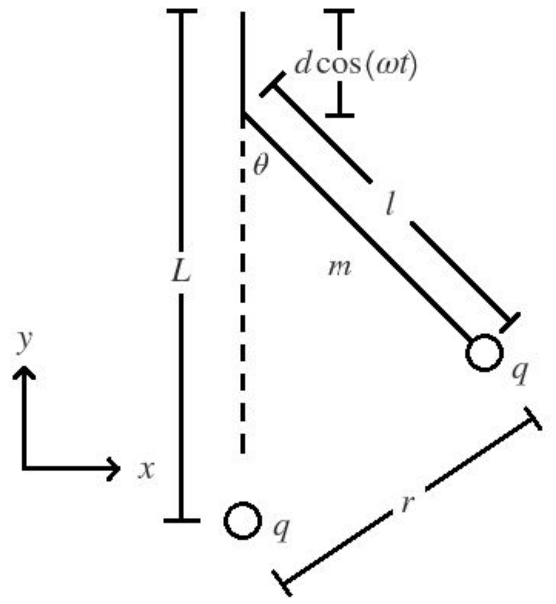

Fig 5: Charged pendulum theoretical set-up

Methodology

1. Lagrangian mechanics and rapid-oscillation averaging was used to derive the equations of motion which were then modified for experimental parameters. [2]
2. Numerical simulations were performed to predict experimental data and stability (See *Fig 1-3*).
3. Numerical experiments were performed to test more complex systems *i.e.* circular oscillations and charged oscillations.
4. Experiments were performed to test the validity of the numerical results. Refer to the Experiment Procedure section.

Results

1. Numerical experiments suggest the charged pendulum can oscillate between its two stable points at high frequencies.
2. Experimental data for arbitrary angle pendulum consistently undershot the numerical results. Refer to *Fig 7*.
3. Potential for significant systematic errors including:
   - Asymmetric Dampening
   - Conservation of Energy
   - Incomplete Model (i.e. Base Size)

Conclusions

- Forced stability of the arbitrary angle pendulum requires high frequencies and is affected by parameters
- Charge pendulum could be explored further for a full analogy to the ion trap or other physical systems

Experiment Procedure

1. Measure length of base, length of pendulum arm, and mass of pendulum arm.
2. Assemble mount and attach Ryobi Jigsaw. Attach base to jigsaw and begin oscillations.
3. Using high-speed camera, capture frequency required for stability.
4. Measure angle of stability with protractor.
5. Compare experimental data to theoretical calculations for calculation of error.

Fig 6: Arbitrary angle pendulum experimental set-up

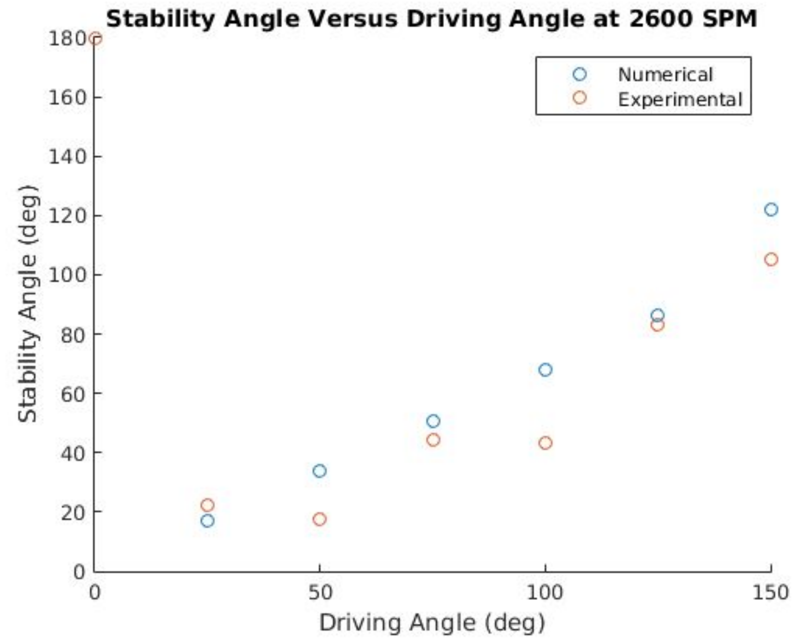

Fig 7: Theoretical stability points and experimental stability points

References

1. P. L. Kapitza, Soviet Phys. JETP 21, 588 (1951).
2. G. J. VanDalen, (2003), 10.1119/1.1603269.

Acknowledgments

This project was mentored by Dr. Ildar Gabitov, whose help is acknowledged with great appreciation. Special thanks to the University of Arizona Department of Mathematics for providing this research opportunity.