Olbers' Paradox

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The Paradox

- In the case that the universe is static and homogeneous, as well as populated by an infinite number of stars, any line of sight from the human eye should eventually intercept the surface of a star. Hence, the night sky should be completely illuminated.

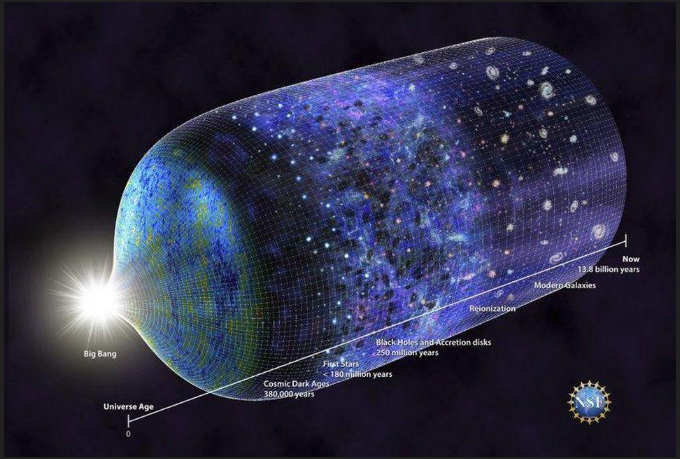
Our Goal

- To analyze historical and modern solutions to Olbers’ Paradox and to analyze the methodology that resulted in their success and failure.

- Also; to simulate a solution to the ‘Dark Night Sky Riddle’ using experimental data and compare our results to previous solutions.

Background/History

- Formulated by Thomas Digges in the 16th century and was discussed by mathematicians and astronomers throughout the 17th and 18th centuries
- First discussed in print in 1722 by Edmund Halley, whose solution involved apparent luminosity dropping by a factor of (1/r^4)
- Other solutions such as the redshifting of cosmic radiation and a finitely old universe have been postulated as well

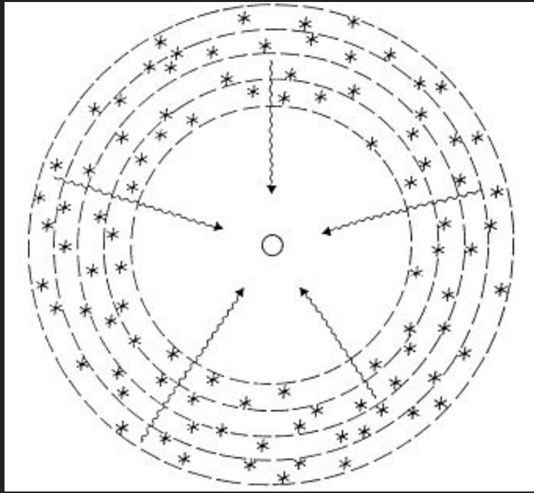


An Interpretation Of the Paradox

- Assumption: Stars have a uniform distribution density in the universe
- Question: What happened to the missing starlight?
- Contributors under this assumption:
  - Edmund Halley
  - Jean-Phillipe Cheseaux
  - Heinreich Olber

Halley’s Argument

- Divide space into spherical concentric shells
- A shell of radius ‘q’ has volume $\Delta V=4\pi q^2 \Delta q$, the volume of the shell increases by q^2 as q increases and so does the number of stars
- The light from each star in a shell decreases by $1/q^2$
- Since consecutive shells have q^2 as many stars with each star contributing $1/q^2$ as much light, each shell contributes same amount of light



Cheseaux's Argument

- Dark spots in sky is simply starlight absorbed by interstellar medium
- Let D be the average distance between stars, $V=D^3$ (average volume occupied by single star
- Surface area of star: $S=\pi R^2$
- Then $\lambda=V/S$ is the distance of background stars (mean free path of light from emission to absorption

Olber's Argument

- Applied both Halley's and Cheseaux's method
- a (Number of stars in a shell) = $n*4\pi q^2 dq$, where $n = 1/D^3$ is stars per volume
- Fraction of sky covered by stars in a shell $d\alpha$= (number of stars)*(fraction of area each star takes up on surface area of sphere) = $a * S/(4\pi q^2) = (S/D^3)dq=(1/\lambda)dq$
- By integrating $d\alpha$ from 0 (observer) to radius $q=r$, we get $\alpha = (r/\lambda)$ the fraction of sky covered by stars up to $q=r$
- Note that when $r=\lambda$, the sky should be completely cover by stars

Olber's Argument (continued)

- This must mean that any star where $r>\lambda$, must be obscured by the stars at $r<\lambda$
- Olbers applies Cheseaux by taking this occultation into account by including a factor of $e^{(-q/\lambda)}$, $d\alpha=e^{(-q/\lambda)}*(1/\lambda)dq$.
- Now the fraction of sky cover by stars due to each shell is less as q increases
- With occultation taken into account and after integrating the new equation from $q=0$ to $q=r$, the fraction of sky covered by stars is $\alpha=1-e^{(-r/\lambda)}$
- Note $\alpha=1$ when $r>>\lambda$

Olber's Argument (continued)

- Much like taking occultation into account, just like Cheseaux, Olber took interstellar absorption into account by including a factor of $e^{(-q/\mu)}$, where μ is the absorption distance the $d\alpha=e^{(-q/\lambda)}*e^{(-q/\mu)}*(1/\lambda)dq$.
- After integrating as before, $\alpha=\mu/(\lambda+\mu) * (1 - e^{(-r(\mu+\lambda)/(\lambda\mu))})$, fraction of sky cover from $q=0$ to $q=r$.
- Note for $r \gg (\lambda\mu)/(\mu+\lambda)$, $\alpha=\mu/(\lambda+\mu)$. In the case of an infinite universe the night sky is not completely covered

Another Interpretation Of the Paradox

- Assumption: The sky is not uniformly covered even in an infinite universe containing an infinite number of stars
- Question: What happened to the missing stars?
- Contributors under this assumption:
  - John Herschel

Herschel’s Argument

- The dark gaps in the sky is due to lack of stars in that line sight, dark gaps are just empty space. That’s IT!
- Herschel also proposed that if an interstellar medium absorbs light it would heat up and would emitted again (thus discounting the absorption factor taken into account by Chesaux and Olber)

Visible Universe Argument

- Assume the universe is unbounded, static and of finite age
- To the observer the visible universe is a radius of the age of the universe ,t, times the speed of light ,c.
- Stars that are not visible in the night sky are stars of light that simply have not reached us yet
- If $\lambda < ct$, the sky is covered by stars. If so, what happened to the missing starlight (this is first interpretation of paradox)
- If $\lambda > ct$, stars do not cover the sky. If so, what happened to the missing stars (this is the second interpretation)

Kelvin’s Approach

- Kelvin came up with an equation relating the energy density in space (u) to energy density on surface of a star (u*): $u=u^*(1-e^{(-ct)/\lambda})$
- If we divide by u* and set r=ct, we obtain the fraction of sky cover by stars $\alpha = (u/u^*)$
- We can see from this equation, the night sky is covered by stars, $\alpha=1$, when the energy density in space equals the energy density on surface of stars, in other words when the visible universe is in thermodynamic equilibrium

Examining Previous Solutions

- Kepler: Concentric Shells (1610)

  - Only true when the universe is finite in size

- Olber: Dust Cloud (1823)

  - The dust cloud would begin to radiate.

- Bondi:Red Shift (1955)

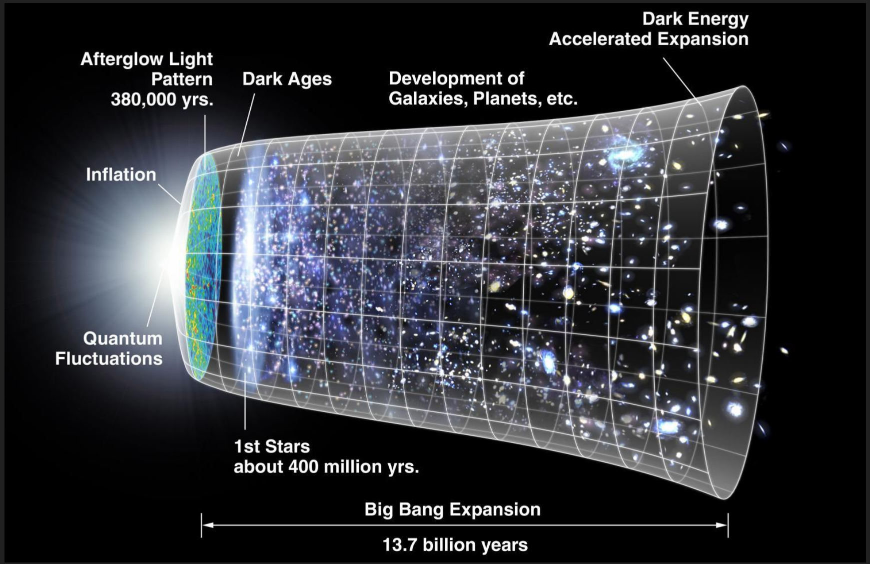
  - Plausible solution! Only in the case of a steady state universe.

The Big Bang Solutions

The Big Bang Solution is the most widely agreed upon plausible solution of Olbers’ Paradox. [3]

- Galaxies beyond the hubble sphere exist outside the temporal boundary of our sight and cannot be observed.

- By the time light from distant galaxies reach us, the galaxies nearest us will grow cold and dark.



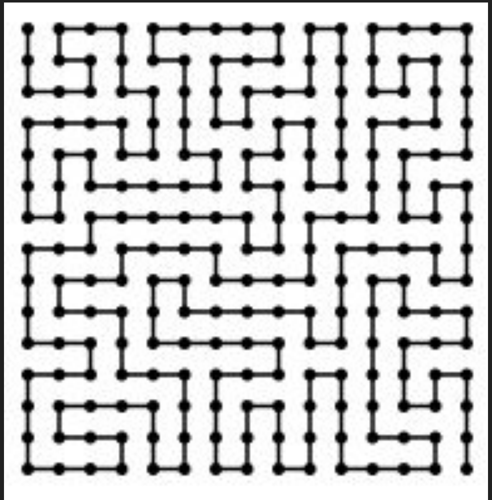
More Complicated Solutions?

There exist a number of plausible solutions to Olbers' Paradox, one of the most interesting is that the formation of new universes accounts for the thermal disequilibrium first witnessed by Kepler, Olbers, and Bondi. [2]

Our Solution: 3-D Self Avoiding Random Walk

Self Avoiding Random Walks

- The self-avoiding random walk theory gives a plausible solution to the paradox
- A self-avoiding random walk is a sequence of moves on a lattice path that does not visit the same point more than once
- Self Avoiding Walks are found in physical systems like polymers chains, that can serve as a model for the distribution of stars in the vacuum of space [1]



The Self Avoiding Random Walk Model

- The application of the S.A.W. model would support Herschel's argument of stars not being uniformly distributed among the universe. Just like in polymers, we model the distribution of stars in a 3-D lattice.
- The observer here is considered the origin and X_0 the closest star to observer
- Each step taken on the 3-D lattice has a unit length and each new step is taken in a direction along an axis to an unoccupied vertex in the lattice. Each vertex represents a sphere (star) with radius *a.*
- The radii of these spheres should be chosen to reflect the actual ratio between the average size of a star and the average distance between two nearby stars

The Self Avoiding Random Walk Model (continued)

- Let W be a self-avoiding random walk of infinite length so that its vertices may be labelled as X_0, X_1, X_{-1}, X_2, X_{-2}, ..., and so on, such that X_k and X_{k+1} are adjacent to each other.
- We will assume that the distribution of X_k can be approximated by the classical Gaussian distribution
- The observed area of star at X_k is

$$\frac{4\pi a_k^2}{|X_k|^2}$$

- The density function of X_k may be approximated by the function:

$$f(X_k) \approx \Big(\frac{1}{\sqrt{2\pi}\sigma_k}\Big)^3 \exp\Big(-\frac{|X_k|^2}{2\sigma_k^2}\Big).$$

The Self Avoiding Random Walk Model (continued)

- We can then integrate the product of observed area of star and density function at X_k over the volume of space and find that the mean contribution of X_k to the total observed area of the observer can be approximated by $4\pi^2 a_k^2/\sigma_k^2$
- The sum of each contribution is the total observed area

$$\sum_{k=\pm1,\pm2,\ldots} \frac{4\pi^2 a_k^2}{\sigma_k^2}.$$

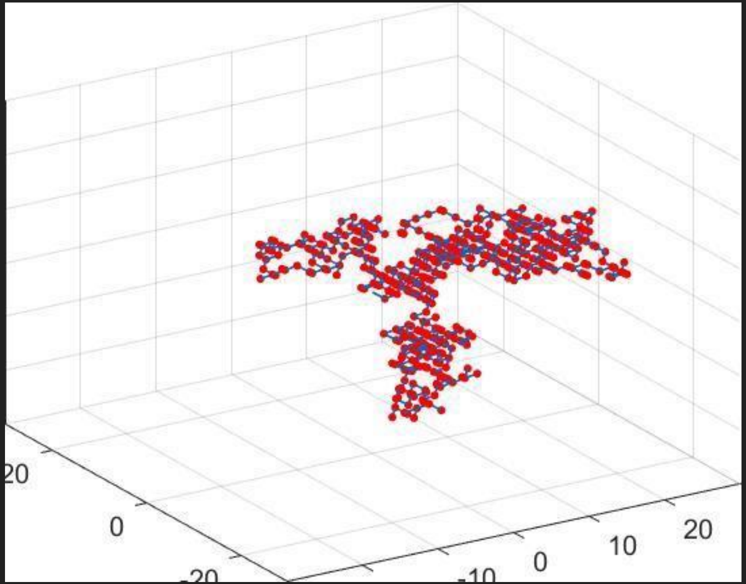
- If we substitute *a* with the radius of the largest known star (in terms of astronomical units), the bound is much smaller than the observer's sky area.
- Conclusion: if the stars follow the typical distribution of the vertices of a self-avoiding equilateral random walk of infinite length, then the total observed area is quite small. Hence, space is assumed to be unlimited and the number of stars finite.

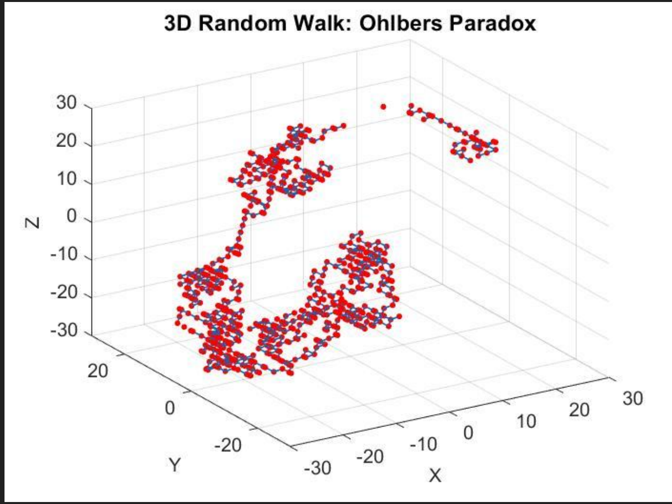
Validity of The Self Avoiding Walk Model

- Again one thing that makes Olber's Paradox paradoxical is the question of whether the universe is infinite or not?
- In the S.A.W. model we assume an infinite (since we integrate over an infinite vacuum of space) static universe with normal distribution which leads to only a fraction of sky covered by stars observed
- Therefore, this model also works for a finite universe since we would get a even smaller fraction due to the smaller limit of integration of the density function. Moreover, it also works for an expanding universe or finite age universe, inclusively.

Simulating the S.A.W.

To further explore this particular solution we modeled a walk containing 1000 stars in a single chain, given the average distance between them was one parsec.




3D Random Walk: Ohlbers Paradox
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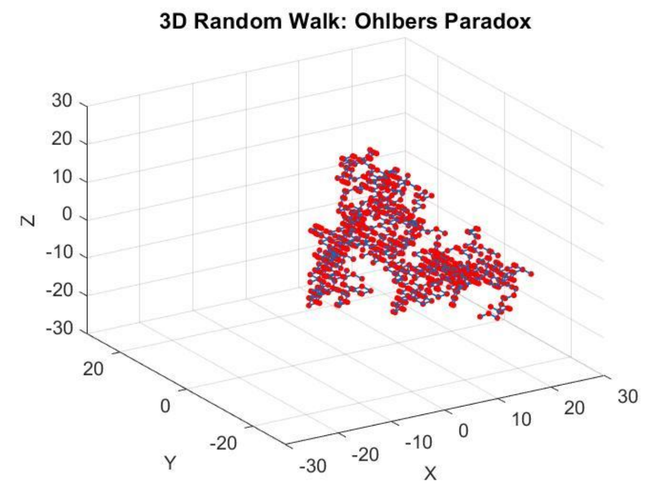
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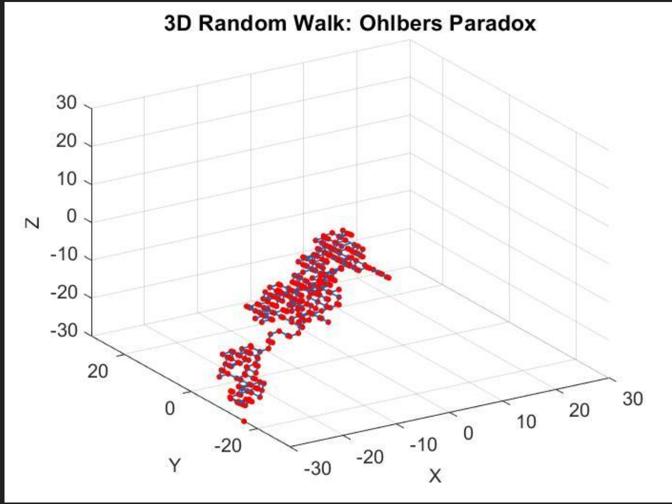
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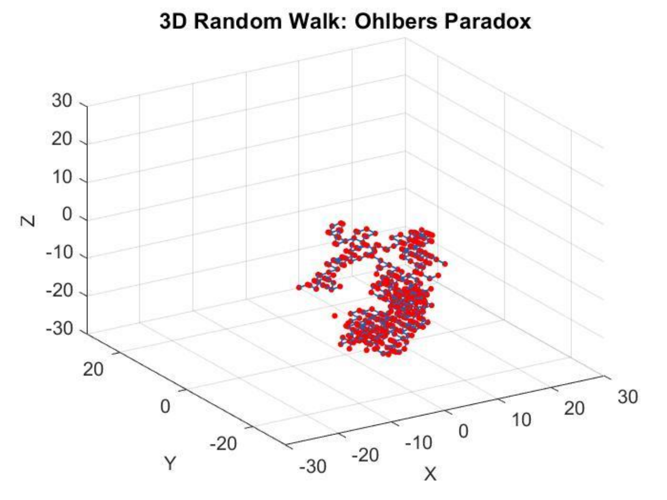
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Analyzing our Solution

- To simplify our solution we bound our answer using the expression:

$$\propto \leq \frac{10^3 \pi a^2}{.17}$$

Where “∝” is the percentage of the night sky covered with stars and “*a”* is the radius of largest known star.

- In a research paper published in 2007, Yuanan Diao found the upper bound for this value to be:

$$\propto = 1.51*10^{-6} = 1.51*10^{-4}\ \%$$

Analyzing our Solution (continued)

- Diao calculated his upper bound using the radius of the largest star known at the time. To replicate these results in our simulation we used the radius of the largest star observed currently, IRAS 05280-6910, to calculate our upper bound [5]:

$$\propto = 2.84*10^{-5} = 2.84*10^{-3}\ \%$$

- (Given, a = 1,738 Solar Radii)

Assessing Our Solution

- Using astronomical observations and approximations for the Mean Free Path of the universe we can calculate the percentage of the sky covered with stars experimentally to be:

$$\propto \sim 10^{-14} - 10^{-17}\ \%$$

Diao's Upper Bound	Our Upper Bound
$1.51*10^{-4}$ %	$2.84*10^{-3}$ %

Extending our Solution: Hubble Sphere

- To improve the accuracy of our solution we apply a new constraint to the problem. We will no longer consider a steady-state static universe of infinite extent, we will now take the universe to be finite spatially and temporally.

$$\sum_{k=\pm 1,\pm 2,\ldots} \frac{4\pi^2 a_k^2}{\sigma_k^2}.$$

α = 2.44*10^{-13} %

Assessing Various Solutions

Method	Percentage of Night Sky Covered
S.A.W. (Us)	$2.44*10^{-13}$ %
Olber	10^{-13} %
Bondi	10^{-13} %
Big Bang	10^{-13} %
Experimental Solution:	10^{-14} - 10^{-17} %

Conclusions

The S.A.W. method we simulated is a viable solution to Olbers paradox though it is not the most plausible when isolated. The next step in this project would be to combine our S.A.W. method with the Big Bang Solution to see how the two behave together.

References

[1] Diao, Y., and A. Stasiak. “Self-Avoiding Random Walks and Olbers' Paradox.” International Journal of Contemporary Mathematical Sciences, vol. 2, 2007, pp. 445–449., doi:10.12988/ijcms.2007.07038.

[2] Frieden, B. R., & Gatenby, R. A. (2019). Spontaneous Formation of Universes from Vacuum via Information-induced Holograms. *arXiv preprint arXiv:1909.11435*.

[3] Friedmann, Aleksandr. “125. On the Curvature of Space.” *A Source Book in Astronomy and Astrophysics, 1900–1975*, 1922, pp. 838–843., doi:10.4159/harvard.9780674366688.c134.

[4] Harrison, Edward. “The Dark Night-Sky Riddle, ‘Olbers's Paradox.’” *Symposium - International Astronomical Union*, vol. 139, 1990, pp. 3–17., doi:10.1017/s0074180900240369.

[5] SCHRÖDINGER, E. Mean Free Path of Protons in the Universe. Nature 141, 410 (1938). https://doi.org/10.1038/141410a0