The mysteries of the evolution of the Earth-Moon system

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Outline

1. Literature
2. Application Background
3. Goldreich Model
4. Methodology
5. Implementation and Results
6. Summary
7. Plan for Final Report/Further Reading

1. Literature

The primary papers we used for the construction of our model are:

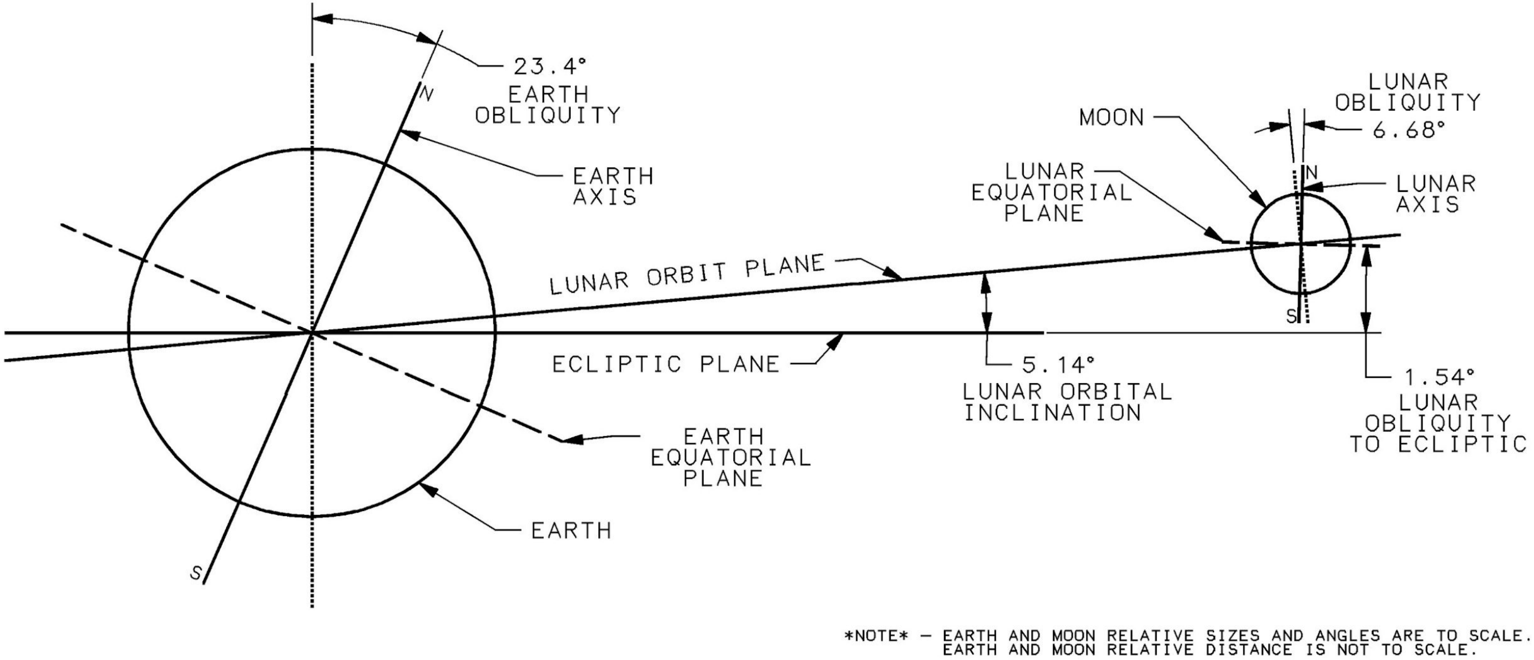
- *History of the Lunar Orbit* by Peter Goldreich 1966
- *Evolution of the Earth-Moon System* by Jihad Touma and Jack Wisdom 1994

2. Application Background

- **The Mystery**: The motion of the Moon is no longer along its original orbit due to the tidal friction in the Earth-Moon system, a process that transfers angular momentum from the Earth's spin to the lunar orbit
- The effect of tidal friction also brings up a question with a seemingly counterintuitive answer: with the effect of tidal friction, energy within the system should be lost, and if energy is being lost, then **why is the moon moving further away from the Earth?**

Modeling the system will be the key to unraveling this mystery!

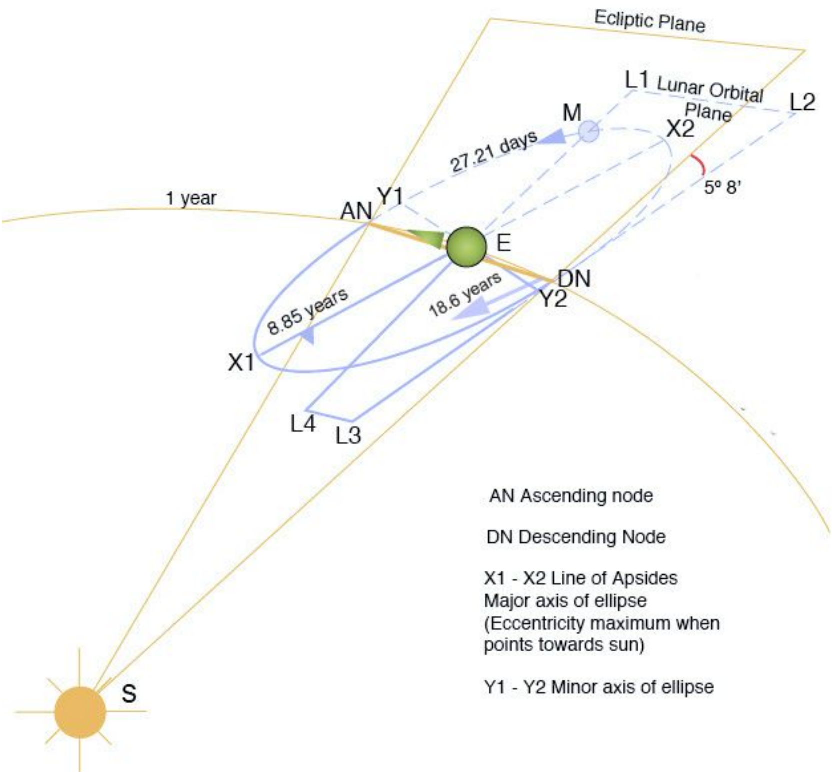
2. Application Background



2. Application Background cont.

The Earth-Moon system is difficult to completely model due to effects from:

- Disturbing potential from other planets
- Tidal friction from Earth/Moon
- The sun’s effect on the system
- Imperfections in their own orbits
- Large magnitude of numbers involved



3. Goldreich Model

Three time scales:

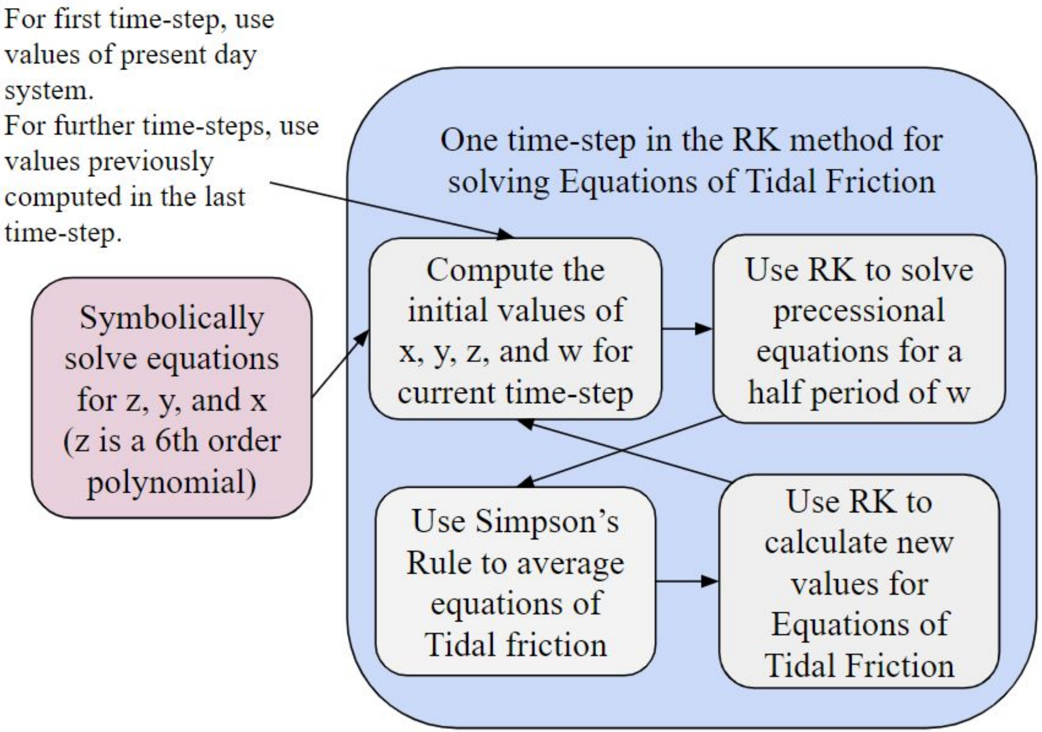
- Orbital (short, ~1 month)
- Precessional (intermediate, 1000s of years)
- Tidal (long, 1+ million years)

Precessional and Tidal Friction Equations viable on the intermediate time scale.

$$\frac{dx}{dt} = \frac{L}{H} zw$$
$$\frac{dy}{dt} = -\frac{L}{h} zw$$
$$\frac{dz}{dt} = \left(\frac{K_2}{h} y - \frac{K_1}{H} x\right) w$$
$$\frac{dw}{dt} = \frac{L}{H} z(yz - x) - \frac{L}{h} z(xz - y) + \left(\frac{K_2}{h} y - \frac{K_1}{H} x\right)(xy - z)$$

$$\frac{dH}{dt} = T_e \cdot \mathbf{a}$$
$$\frac{dh}{dt} = T_m \cdot \mathbf{b}$$
$$\frac{d\Lambda}{dt} = (T_e + T_m) \cdot \mathbf{c}$$
$$\frac{d\chi}{dt} = \frac{2K_1}{H} x T_e \cdot \mathbf{c} + \frac{2K_2}{h} y (T_m \cdot \mathbf{c} + y T_m \cdot \mathbf{b})$$
$$\frac{da}{dt} = 2a T_{moon} \cdot \mathbf{b}/h$$

4. Methodology



5. Implementation and Results

The initial values of several different components of the system have been computed.

Orbital data has been obtained for the ‘short’ time scale (1 lunar orbit). The average of this data is used for the ‘long’ time scale.

```
# calculate the values using the functions
Hval = Hfun().value
hval = hfun(a).value
Lval = Lfun(a).value
K1val = K1fun().value
K2val = K2fun(a).value
Lamval = Lamfun(a).value
chival = Chifun(a).value
```

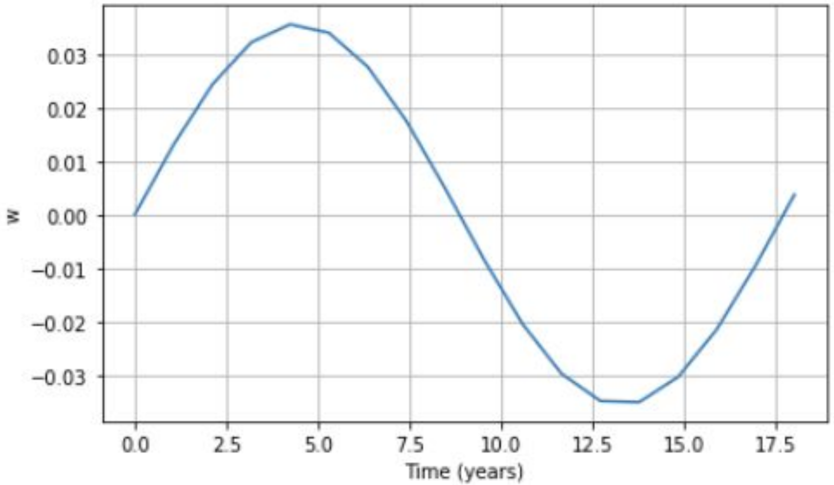
```
7.099346273804928e+33 2.7028960032495545e+34 4.627578397551414e+22 2.1303168480396215e+22 3.233558216518935e+26 3.3435516634428
75e+34 3.208123335840844e+26
```

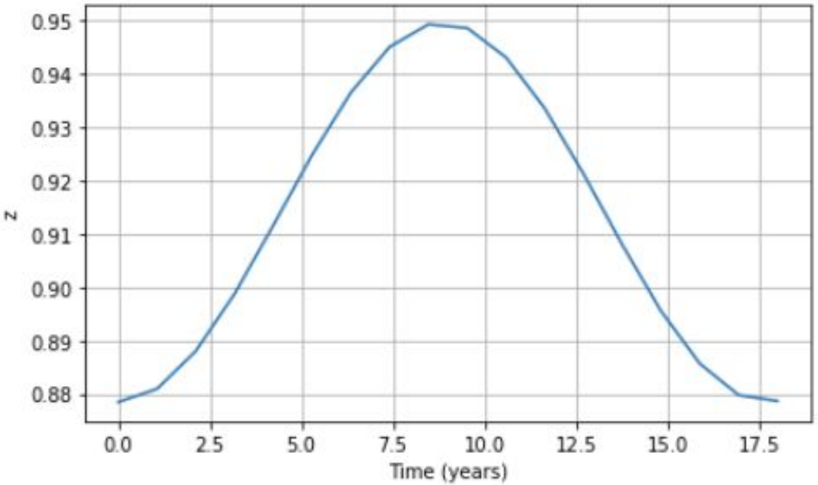
5. Implementation and Results

$$
\begin{aligned}
& z^6 + \frac{z^5\left(-4H^3K_2Lh + 4H^3L^2h - 4HK_1Lh^3 + 4HL^2h^3\right)}{4H^2L^2h^2} \\
& + \frac{\begin{array}{l} z^4\left(H^4K_2^2 - 2H^4K_2L + H^4L^2 + 2H^2K_1K_2h^2 - 2H^2K_1Lh^2 + 4H^2K_2LLam^2 - 2H^2K_2Lh^2 + 2H^2L^2h^2 - 8H^2LXh^2 + K_1^2h^4\right. \\ \left. + 4K_1LLam^2h^2 - 2K_1Lh^4 + L^2h^4\right) \end{array}}{4H^2L^2h^2} \\
& + \frac{\begin{array}{l} z^3\left(-4H^3K_2Lhw^2 + 4H^3K_2Lh + 4H^3K_2Xh - 8H^3LXh - 8HK_1K_2Lam^2h + 4HK_1LLam^2h - 4HK_1Lh^3w^2 + 4HK_1Lh^3 + 4HK_1Xh^3\right. \\ \left. + 4HK_2LLam^2h - 8HLXh^3\right) \end{array}}{4H^2L^2h^2} \\
& + \frac{\begin{array}{l} z^2\left(2H^4K_2^2w^2 - 2H^4K_2^2 - 2H^4K_2Lw^2 + 2H^4K_2L + 2H^4K_2X - 2H^4LX - 2H^2K_1K_2Lam^2 + 4H^2K_1K_2h^2w^2 - 4H^2K_1K_2h^2\right. \\ + 2H^2K_1LLam^2 - 2H^2K_1Lh^2w^2 + 2H^2K_1Lh^2 + 2H^2K_1Xh^2 + 2H^2K_2^2Lam^2 - 2H^2K_2LLam^2 - 2H^2K_2Lh^2w^2 + 2H^2K_2Lh^2 \\ - 4H^2K_2Lam^2X + 2H^2K_2Xh^2 - 4H^2LXh^2 + 4H^2X^2h^2 + 2K_1^2Lam^2h^2 + 2K_1^2h^4w^2 - 2K_1^2h^4 + 4K_1K_2Lam^4 - 2K_1K_2Lam^2h^2 \\ \left. - 2K_1LLam^2h^2 - 2K_1Lh^4w^2 + 2K_1Lh^4 - 4K_1Lam^2Xh^2 + 2K_1Xh^4 + 2K_2LLam^2h^2 - 2LXh^4\right) \end{array}}{4H^2L^2h^2} \\
& + \frac{\begin{array}{l} z\left(4H^3K_2Xhw^2 - 4H^3K_2Xh + 4H^3X^2h - 8HK_1K_2Lam^2hw^2 + 8HK_1K_2Lam^2h - 4HK_1Lam^2Xh + 4HK_1Xh^3w^2 - 4HK_1Xh^3\right. \\ \left. - 4HK_2Lam^2Xh + 4HX^2h^3\right) \end{array}}{4H^2L^2h^2} \\
& + \frac{\begin{array}{l} H^4K_2^2w^4 - 2H^4K_2^2w^2 + H^4K_2^2 + 2H^4K_2Xw^2 - 2H^4K_2X + H^4X^2 - 2H^2K_1K_2Lam^2w^2 + 2H^2K_1K_2Lam^2 + 2H^2K_1K_2h^2w^4 \\ - 4H^2K_1K_2h^2w^2 + 2H^2K_1K_2h^2 - 2H^2K_1Lam^2X + 2H^2K_1Xh^2w^2 - 2H^2K_1Xh^2 + 2H^2K_2^2Lam^2w^2 - 2H^2K_2^2Lam^2 + 2H^2K_2Lam^2X \\ + 2H^2K_2Xh^2w^2 - 2H^2K_2Xh^2 + 2H^2X^2h^2 + K_1^2Lam^4 + 2K_1^2Lam^2h^2w^2 - 2K_1^2Lam^2h^2 + K_1^2h^4w^4 - 2K_1^2h^4w^2 + K_1^2h^4 - 2K_1K_2Lam^4 \\ - 2K_1K_2Lam^2h^2w^2 + 2K_1K_2Lam^2h^2 + 2K_1Lam^2Xh^2 + 2K_1Xh^4w^2 - 2K_1Xh^4 + K_2^2Lam^4 - 2K_2Lam^2Xh^2 + X^2h^4 \end{array}}{4H^2L^2h^2} \\
& = 0
\end{aligned}
$$

5. Implementation and Results

- The values for x, y, z, and w over one period of w are calculated.
- The averages of these values will be used to integrate the tidal friction equations on the long time scale





5. Implementation and Results

```
def dHdt(H, lam, a, chi, t, x, y, z, w, T_earth, T_moon, K1, K2, L, h):
    Tearthdota = T_earth[1]*math.sqrt(1-z**2) + T_earth[2]*z
    dHdt = Tearthdota
    return dHdt

def dlamdt(H, lam, a, chi, t, x, y, z, w, T_earth, T_moon, K1, K2, L, h):
    Tearthdotc = T_earth[0]*(w/math.sqrt(1-z**2)) - T_earth[1]*(x-y*z)/(math.sqrt(1-z**2)) + T_earth[2]*y
    Tmoondotc = T_moon[0]*(w/math.sqrt(1-z**2)) - T_moon[1]*(x-y*z)/(math.sqrt(1-z**2)) + T_moon[2]*y
    dlamdt = Tearthdotc + Tmoondotc
    return dlamdt

def dadt(H, lam, a, chi, t, x, y, z, w, T_earth, T_moon, K1, K2, L, h):
    Tmoondotb = T_moon[2]
    dadt = 2*a/h*Tmoondotb
    return dadt

def dchidt(H, lam, a, chi, t, x, y, z, w, T_earth, T_moon, K1, K2, L, h):
    Tearthdotc = T_earth[0]*(w/math.sqrt(1-z**2)) - T_earth[1]*(x-y*z)/(math.sqrt(1-z**2)) + T_earth[2]*y
    Tmoondotc = T_moon[0]*(w/math.sqrt(1-z**2)) - T_moon[1]*(x-y*z)/(math.sqrt(1-z**2)) + T_moon[2]*y
    Tmoondotb = T_moon[2]
    Tearthdotb = T_earth[2]
    Tmoondota = T_moon[1]*math.sqrt(1-z**2) + T_moon[2]*z
    dchidt = ((2*K1/H*x * Tearthdotc) + (2*K2/h*y*(Tmoondotc + y*Tmoondotb)) + \
              2*L*z*(Tearthdotb/H + (Tmoondota - 4*z*Tmoondotb)/h))
    return dchidt
```

6. Summary

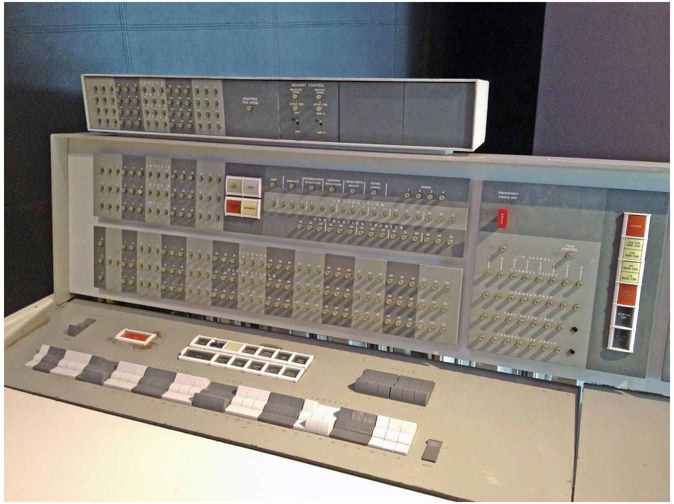
We are in-progress of implementing Goldreich's models within our numerical environment (Python).

At the time of his work in 1966, numerical tools were greatly limited in comparison to what is available today.

These advances in numerical methods provide us the capability to run these models at a level of precision unavailable at the time.

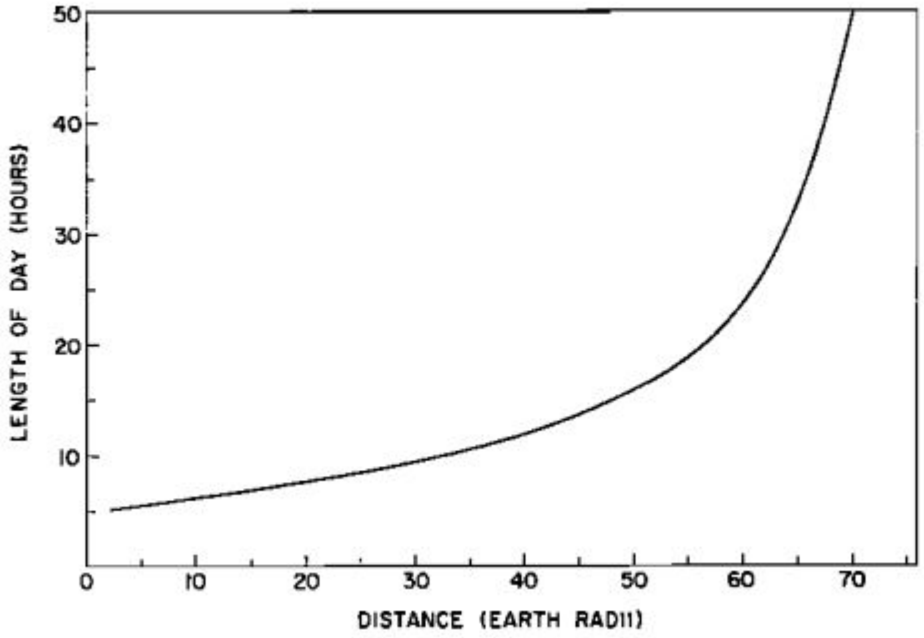
We can compare our modern results to Goldreich's that he offers within his paper.

Goldreich ran his model on an IBM 7094 - each time step on the 'long' scale (1+ million years) took 20 seconds to finish. With modern technology, this can be run both faster and with more precision



6. Summary: Back to the Mystery

- Tidal friction decreases angular momentum of the Earth, this energy is transferred to the lunar orbit, increasing the Earth-Moon distance.
- Eventually, the Earth becomes tidally locked with the Moon, such that one day will become the period of the lunar orbit.



7. Plan for Final Report

1. Complete the implementation of Goldreich’s model within our numerical environment
   a. Implement the Runge-Kutta 4 approach for the long time scale
   b. Run the simulation on the long time scale with high precision
2. Compare numerical results with Goldreich (with what is available)
3. Identify any differences of interest that may have not been present before

Further Reading / References

History of the Lunar Orbit by Peter Goldreich

Evolution of the Earth-Moon System by Jihad Touma and Jack Wisdom

Proterozoic Milankovitch cycles and the history of the solar system by Stephen Meyers and Alberto Milinvernob

MacDonald, G. J. F. (1964), Tidal friction, Rev. Geophys., 2(3), 467– 541, doi:10.1029/RG002i003p00467.

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