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Math 485 Midterm Project Report

Eye-Brain Connection

Abstract: The development of the optical neuron between the eye and the brain have been studied by many people in the past, with the aim of understanding how this connection developed and how its process is encoded in our genetics that can only carry about 1Gb of information to the next generation. In this report we try to understand this connection by modeling the growth of one neuron due to one source of attracting proteins. Eventually going to model it for multiple neurons, multiple sources and finally 3-dimensions.

History:

The first time humans knew that the eye is connected to the brain was discovered by Pliny who lived between 273-70 CE. Then in the second century Galen described how the retina net-like looks like our brains. In 1604, J. Kepler told us that the picture we see with our eyes is built on our retina. Later on, in the 19th Century, J. Muller evolved the law of specific nerve energies such that he illustrated the sense the organ opens only to the nerve impulse that sends the income image to the brain to be analyzed. Then in 1855, B. Panizza wrote his book *Observations on the optic nerve*, and publish it causing the naked eye to be able to see the visual projectiles that are projected from the optic nerve to the cerebral, and hence he was able to claim that the visual function's location is in the visual cortex in the occipital region. After these studies, researchers and specialists continued to discover more and more about the human eye.

Introduction:

To understand how the human eyes work, experts and researchers tried to study it and study its neuron system and how the eyes reflect to the brain what we see and make it recognizable to us. The problem is that the human eyes are more complex than what we think, and the process of studying the problem in the human eyes remain incomplete due to the complexity and the sophisticated system (Kandel et al. 2000). Several approaches were done in order to understand the neuron system in the eye-brain system and relate it to the human's eye system.

Our approach in this project is to understand how the human eye works using Laplacian potential, that represent the concentrations of proteins between the eye and the brain. At first we tried to use the stochastics process. Nevertheless, because it takes more time to achieve our goal, we switched to the potential function and the protein source concentration function in 2D and 3D derived here as well. This way, we will be able to assume where the concentration of the protein is concentrated at, and how the neurites get to develop towards the source of the protein in our 2D model, and make it all the way to the brain so that we achieve the desired connection.

Theory:

The protein source concentration function in 2D

Case 1: 2-Dimensional protein concentration of one source.

The concentrations of specific proteins in the area between the retina and the brain. In the simplistic model below, we first try to find a concentration function for the protein. Illustrated in (figure 1).

To find the concentration, or the potential, function, we assume that this function satisfies Laplace's equation with the following conditions:

1- The solution to L.E is separable. $\psi(r,\theta) = u(r)v(\theta)$
2- The boundary conditions are;
   a) Symmetry with respect to θ. $v(\theta) = v(-\theta)$
   b) the source has a radius $r = \epsilon$ and has an initial concentration of ψ_0 at $r = \epsilon$.
   c) At the boundary where the source is located, the concentration does not penetrate the wall. Or $\frac{d\psi}{d\theta} = 0 \; at \pm \frac{\pi}{2}$
   d) $v(\theta) = v(\theta + 2\pi)$
   e) The concentration decays with distance.

$$\nabla \cdot \nabla\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$$

$$= \left[\frac{\partial}{\partial r}\left(\frac{\partial\psi}{\partial r}\right) + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{\partial\psi}{\partial\theta}\right)\right] = 0$$

After multiplying by r^2 and separating the variables we have two equations:

$$\frac{r^2}{u}\frac{d^2u}{dr^2} + \frac{r}{u}\frac{du}{dr} = \lambda$$

$$\frac{1}{v}\frac{d^2v}{d\theta^2} = -\lambda$$

Where λ is the separation constant.

The solution to the angular part is $v(\theta) = \mathrm{a}\cos\left(\sqrt{\lambda}\theta\right) + b\sin(\sqrt{\lambda}\theta)$ which result in $v(\theta) = a\cos\left(\sqrt{\lambda}\theta\right)$ for the symmetry condition above.

Using condition d), we find that $\lambda = n^2, n \in Z$. Notice that with this result, the condition c), is satisfied as well. Hence we have

$$v(\theta) = a\cos(n\theta)$$

The radial part is solved by assuming that $u(r) = cr^k$, where c and k are constants.

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} = n^2u$$

Applying this function in the differential equation above we find that $k = \pm n$. Thus $u(r) = D\,r^n + Cr^{-n}$, using condition e) above we have

$u = Cr^{-n}$

Hence our function

$$\psi(r,\theta) = u(r)v(\theta) = a_0 + \sum_{n=1} a_n r^{-n}\cos(n\theta)$$

We keep the term that is least vanishing at large distances for simplicity, keeping in mind that some terms become more efficient at some angles, i.e. at $\theta = \frac{\pi}{2}$, the second term is dominant all the time compared to the first.

$$\psi(r,\theta) \approx \frac{a_1 \cos(\theta)}{r} + a_0$$

$a_0 = \frac{1}{2\pi} \int \psi d\theta$ from 0 to 2π, or simply the initial concentration at te boundary, $\theta = \pm\frac{\pi}{2}$.

While $a_1 = \frac{1}{\pi} \int \cos\theta \, \psi d\theta$, from 0 to 2π. Or simply the concentratration at the tip of the source $\psi_0\epsilon - a_0 = a_1$.

Case2: Multiple sources in 2D

Concentration of proteins should add up linearly. However, the initial position of the boundary will be shifted for each function in the y direction. Transforming $\psi(r,\theta)$➔$\psi(x, y \pm ms)$

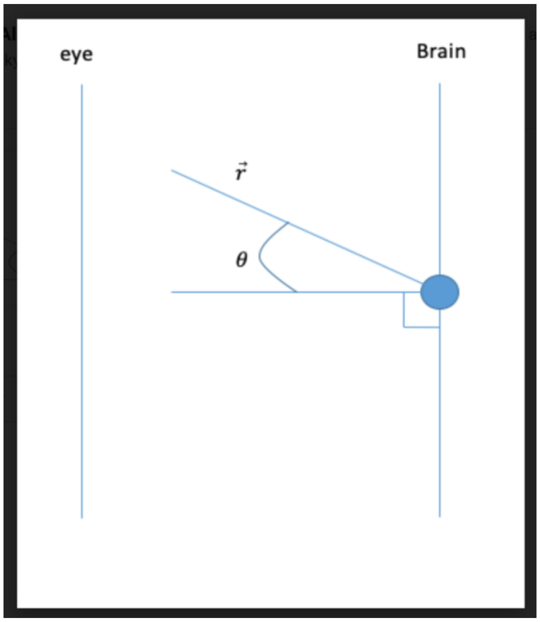
Assuming a constant separation of the sources by a distance $\delta y = m\square$. Where $s = constant$ and $m \in Z$.

$$\psi(x,y)_{total} = a_0 + \sum_{\substack{m=\pm q \; if \; sources \; are \; odd \; in \; number \\ q=0 \\ m=\pm p \; if \; sources \; are \; even \; in \; number \\ p=1}} (\psi_0\epsilon - a_0) \frac{x}{(x^2 + (y + ms)^2)}$$

Where our initial source is at $m = 0$. Then, any additional source will be above or below it symmetrically for an odd number of sources.

We can keep the polar form in the following form

$$\psi(r,\theta)_{total} = a_0 + \sum_{\substack{m=\pm q \; if \; sources \; are \; odd \; in \; number \\ q=0 \\ m=\pm p \; if \; sources \; are \; even \; in \; number \\ p=1}} (\psi_0\epsilon - a_0) \frac{r\cos(\theta)}{(r^2 + msrsin(\theta) + (sm)^2)}$$

Figure 1) 

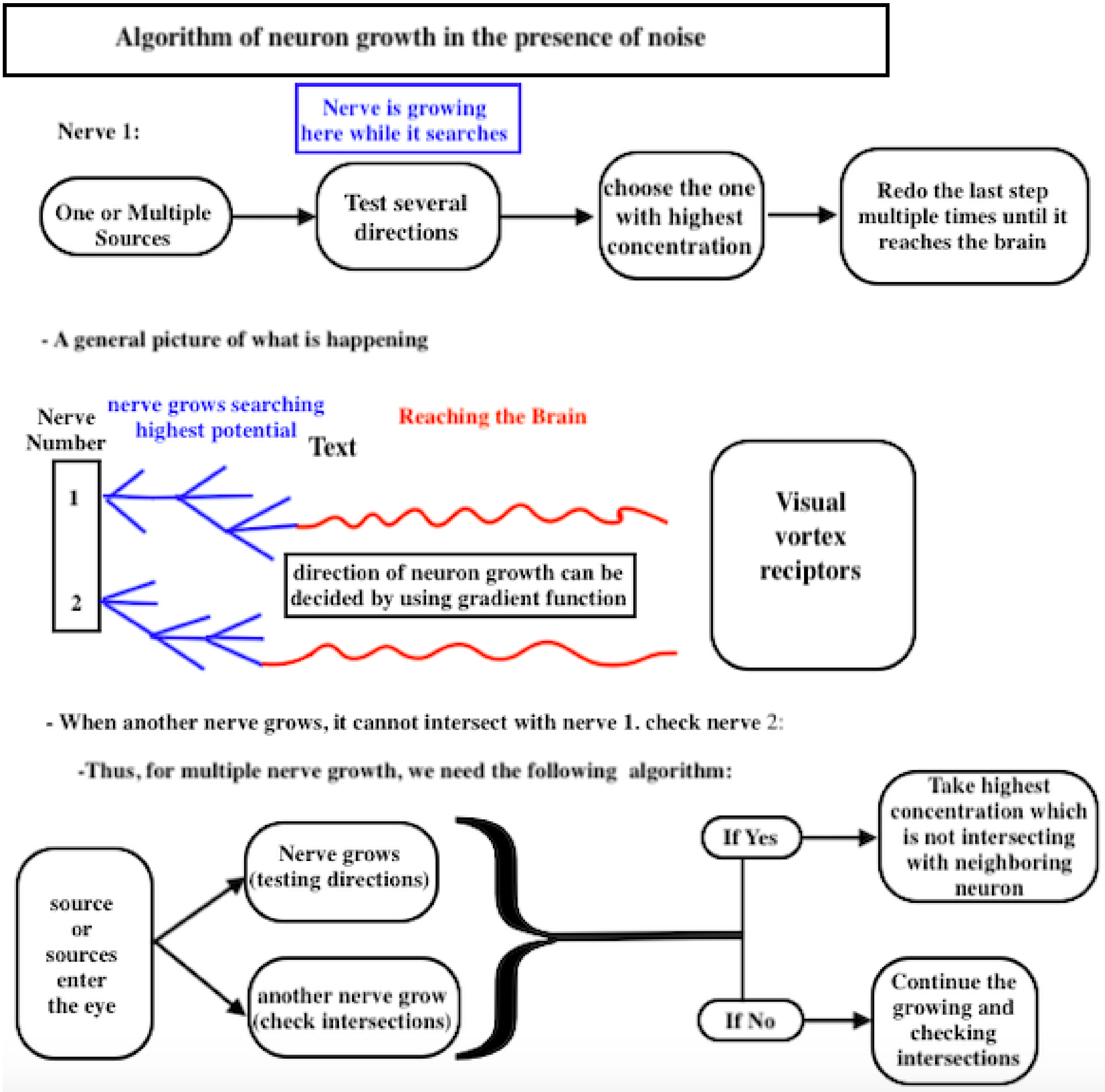
The chemical potential is directly related to the direction of the growth of neurons. By finding the gradient of the potential and the norm of this gradient we can determine the growing direction for the neurons.

The gradient of this potential for a single source in polar coordinate should be:

$$\frac{\vec{\nabla}\psi(r,\theta)}{\left|\vec{\nabla}\psi(r,\theta)\right|} = growth\ direction = -\cos(\theta)\,\hat{r} - \sin(\theta)\,\hat{\theta}$$

This is how we determine the direction of neuron growing due to one single source.

Algorithm

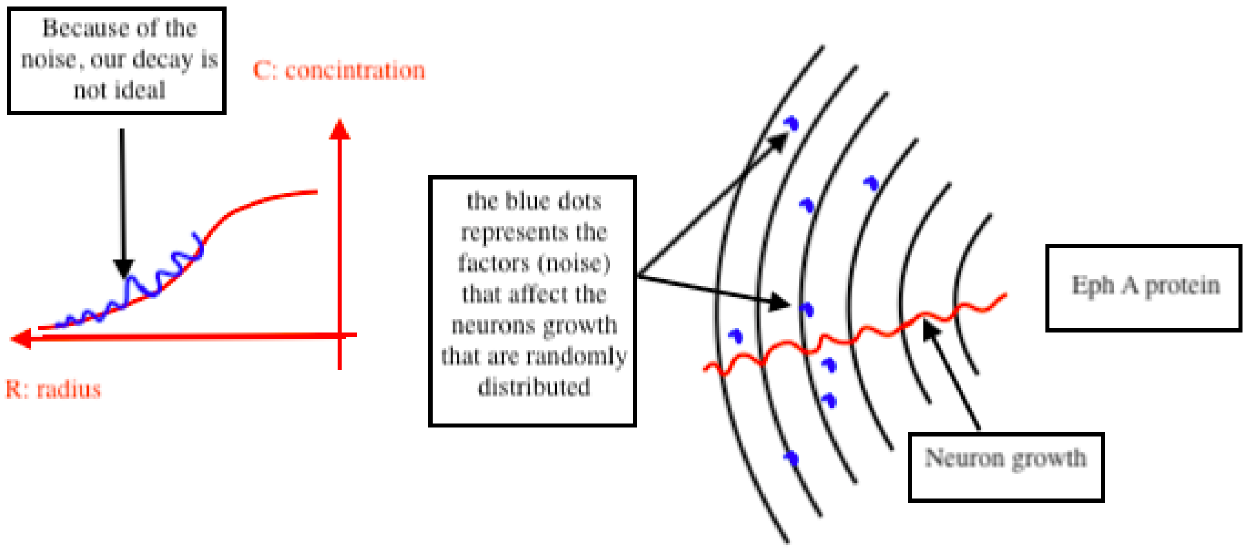

Algorithm of neuron growth in the presence of noise
Nerve 1:
Nerve is growing here while it searches
One or Multiple Sources
Test several directions
choose the one with highest concentration
Redo the last step multiple times until it reaches the brain
- A general picture of what is happening
Nerve Number
nerve grows searching highest potential
Text
Reaching the Brain
1
2
direction of neuron growth can be decided by using gradient function
Visual vortex reciptors
- When another nerve grows, it cannot intersect with nerve 1. check nerve 2:
-Thus, for multiple nerve growth, we need the following algorithm:
source or sources enter the eye
Nerve grows (testing directions)
another nerve grow (check intersections)
If Yes
Take highest concentration which is not intersecting with neighboring neuron
If No
Continue the growing and checking intersections

Why there is not a smooth line from the nerve when it reaches the brain?

Assuming we have protein Eph A (which is responsible in guiding the neurons from eye to brain) with the following distribution:

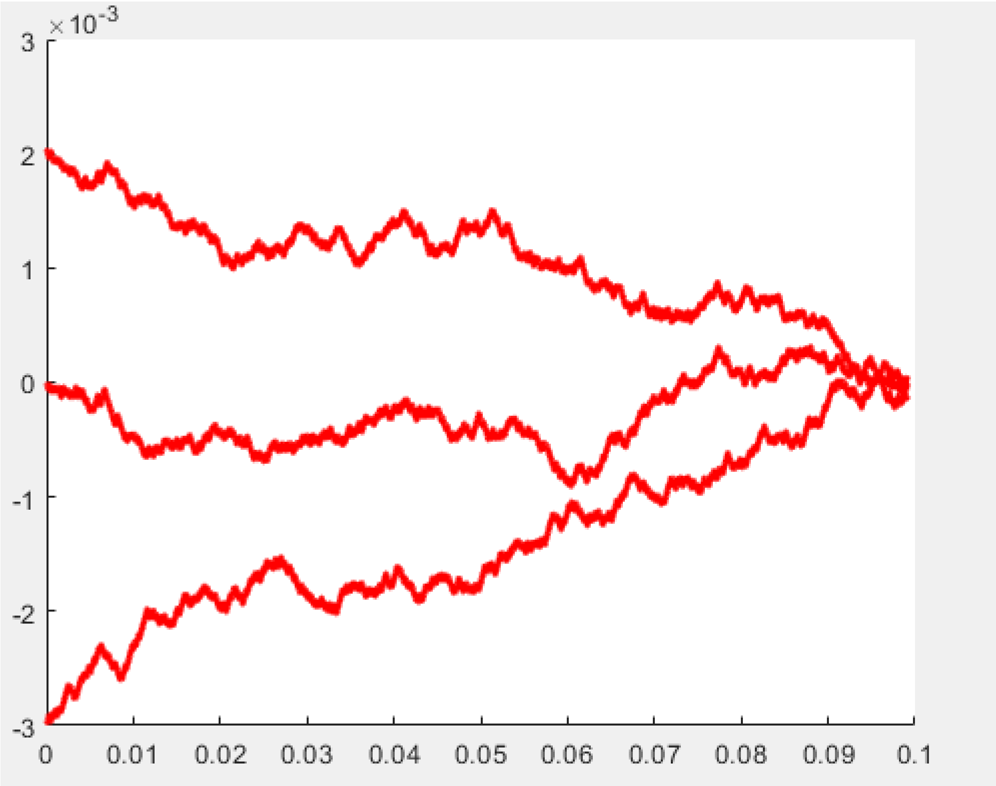
A decying distribution

because there are some factors, name them noise, that affect the neurons growth, then:



Coding the algorithm:

After coding the algorithm, we get the following:



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This figure shows the track of neuron's growth in a single source case, three neurons are located at a different position in y-direction but same position in the x-direction, and finally, all of them will tend to the source.

Code and results:

```
source_r = 1 * 10^-3;
neuron = [0 -0.003];
step_size = 1*10^-4;  %changed size bigger to lower the running time
```

```
%a0,a1=1;
source = [0.1,0];
psi_0 = 1;
n = 0; %record number of movement.
hold on;
while true
%     if n == 10
%         break;
%     end
    theta = get_theta(neuron,source); %get the angle of move
    new_neuron = move(neuron,step_size,theta); %move the neuron
    final_neuron = make_noise(new_neuron,step_size); %make noise
    neuron = final_neuron; %update the neuron
    if_reach = check_reach(neuron,source,source_r);%check if reach
    plot(neuron(1),neuron(2),'r.'); %record the position of neuron
    if if_reach == true %if neuron reach the source and its radius
        break; %end the growth,else, keep going.
    end
    n = n+1;
end
hold off;
```

This code snippet above simulates a neuron's growth in a single source case. And the whole function is also called other functions to simulate the total process. In the growing process of a neuron, it needs to determine the direction that it wants to grow, make growth, and, influenced by noise, repeat the process above until it meets the source. Here, in our program, each step has its own function, the language used for our project is Matlab.

```
function [theta] = get_theta(neuron,source)
if neuron(2) - source(2) == 0
```

```
    theta = 0;
else
    x_dis = abs(neuron(1) - source(1));
    y_dis = abs(neuron(2) - source(2));
    theta = atand(y_dis/x_dis);
end
```

This function determines the direction that the neuron wants to grow towards. The direction was determined directly by the position of source to the current position of the neuron. The direction is 0 if the neuron and source's y-position are the same.

```
function new = move(original,step_size,theta)
if original(2) > 0
    y = -cosd(90-theta)*step_size;
else
    y = cosd(90-theta)*step_size;
end
x = sind(90-theta)*step_size;
new = [original(1)+x original(2)+y];
```

This function grows the neuron with the given direction it wants to grow(theta), and the step size which means the total length it grows every time. Here if the neuron's current position in y is higher than the source, it will move down, and if it is lower than the source, it will move up.

```
function [final_move] = make_noise(move,step_size)
final_move = move;
num = randi(100);
if num<=50
    final_move(2) = move(2) - step_size/4;
else
    final_move(2) = move(2) + step_size/4;
end
final_move = final_move;
```

We know the neuron’s growth will be influenced by the noise, so here I simulate the random noise will happen to the neuron, it may higher or lower the y-position of the neuron, and the possibility for both cases are the same, the noise is a quarter of the total movement.

```
function if_reach = check_reach(neuron,source,source_radius)
distance = sqrt((neuron(1)-source(1))^2 + (neuron(2)-source(2))^2);
if distance > source_radius
    if_reach = false;
else
    if_reach = true;
end
```

This function checks the linear distance from the neuron to the center of the source, if the distance is smaller than the radius of the source, that means the neuron already touches the source, the growth of the neuron will stop, return true value here, otherwise return false and the growth keeps going.

```
function [psi] = single_source(r,theta)
psi = 1 + cos(theta)/r;
```

This is the function of Laplace’s equation in a single-source case, we already talk about the equation above, here we assume the a0 and a1 to be 1, so the equation looks not very confusing. With a deeper step in this project, we will give Laplace's equation for multiple sources’ algorithm and results.

Work Cited

http://arthistoryresources.net/visual-experience-2014/eye-to-brain.html

Schluppeck, D. (2016, October 12). Science still battles to understand the human eye. Retrieved April 14, 2020, from https://www.ft.com/content/3f2eb926-654c-11e6-8310-ecf0bddad227

Department of Biochemistry and Molecular Biophysics Thomas Jessell, Siegelbaum, S., & Hudspeth, A. J. (2000). *Principles of neural science* (Vol. 4, pp. 1227-1246). E. R. Kandel, J. H. Schwartz, & T. M. Jessell (Eds.). New York: McGraw-hill.