Mysteries of the Earth-Moon System

Project Description
- Many mysteries surround the behavior of the Earth-Moon system.
- A primary source of this mystery is that the motion of the Moon is no longer along its original orbit due to the presence of tidal friction.
- Goldreich's 1966 paper on this system is classic, and details transfers of angular momentum from the Earth's spin to the lunar orbit [1].
- His work and others ask the question that with the effect of tidal friction, energy within the system should be lost - and if energy is being lost, then one would expect the Moon to be falling into the Earth, however, observations show the Moon is moving further away from the Earth, so why is this the case?
- This question is the major force driving our study and analysis of Goldreich's models of the Earth-Moon System.
- Our goal was to implement the models formulated by Goldreich with modern computational capability, to investigate the system to a level of precision unavailable to Goldreich.

Scientific Challenges
- The Moon-Earth system is incredibly complex and thereby impractical to calculate analytically entirely. The creation of a numerical model allows researchers to project the behavior of the Moon in its future, paving way for work in the area of tides, sedimentation rates, and more [3].
- Challenges include calculating system parameters both for initial values and within each time-step.

Potential Applications
- Proterozoic Milankovitch Cycles [3]
- Prediction of Earth’s future tide behavior and day length
- Modeling the future behavior and environment of our solar system

Methodology
1. The model was derived following Goldreich's approach. First, disturbing potential equations are described and averaged over the short time scale to derive equations for secular torque. These are used to derive a set of precessional equations and constants valid on the intermediate time scale. tidal friction equations derived by MacDonald [4] are used to augment the precessional constant equations, H, h, \( \Lambda \), and \( \chi \) for a full set of tidal friction equations.
2. The numerical methods used to solve the equations for H, H, \( \Lambda \), and \( \chi \) were run in Python. A four step Runge-Kutta was used, where for each time step the tidal friction equations were re-averaged with the new constant values. To average the equations the precessional equations were initialized then solved with another Runge-Kutta, then a Simpson's rule was used for the integration over these values.

Glossary of Technical Terms
Disturbing Potential: The orbital motion of the Earth resulting in a varying energy potential of the Earth-Moon system
Short/Intermediate/Long Time Scale: Different periods of integration for the Earth-Moon system, to allow for feasible analytical and numerical integration. The periods are 1 month-1 year, 18-1000s of years, and millions of years.
Secular Torque: Torques that do not average to zero over an orbital period.
Proterozoic: Induced rotational motion of a spinning body's axis by torque on the rotational axis.

Results
- The equations of Tidal Friction were solved using equippaced time-steps over a scale of 10 billion years to analyze the changes in the system.
- Our data thereby grants us insight on how Earth's spin, the Earth-Moon distance, and their momentum will change over time or their past values.
- Used Macdonald Tides, averaged precessional changes without Simpson's Rule, Used Touma-Wisdom model to correct assumptions by Goldreich.

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References

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Diagram depicting some of the critical radii, angles, and vectors necessary in the modeling of the Earth-Moon System.

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