Double Inverted Pendulum on Vibrating Base Dynamics

Project Description

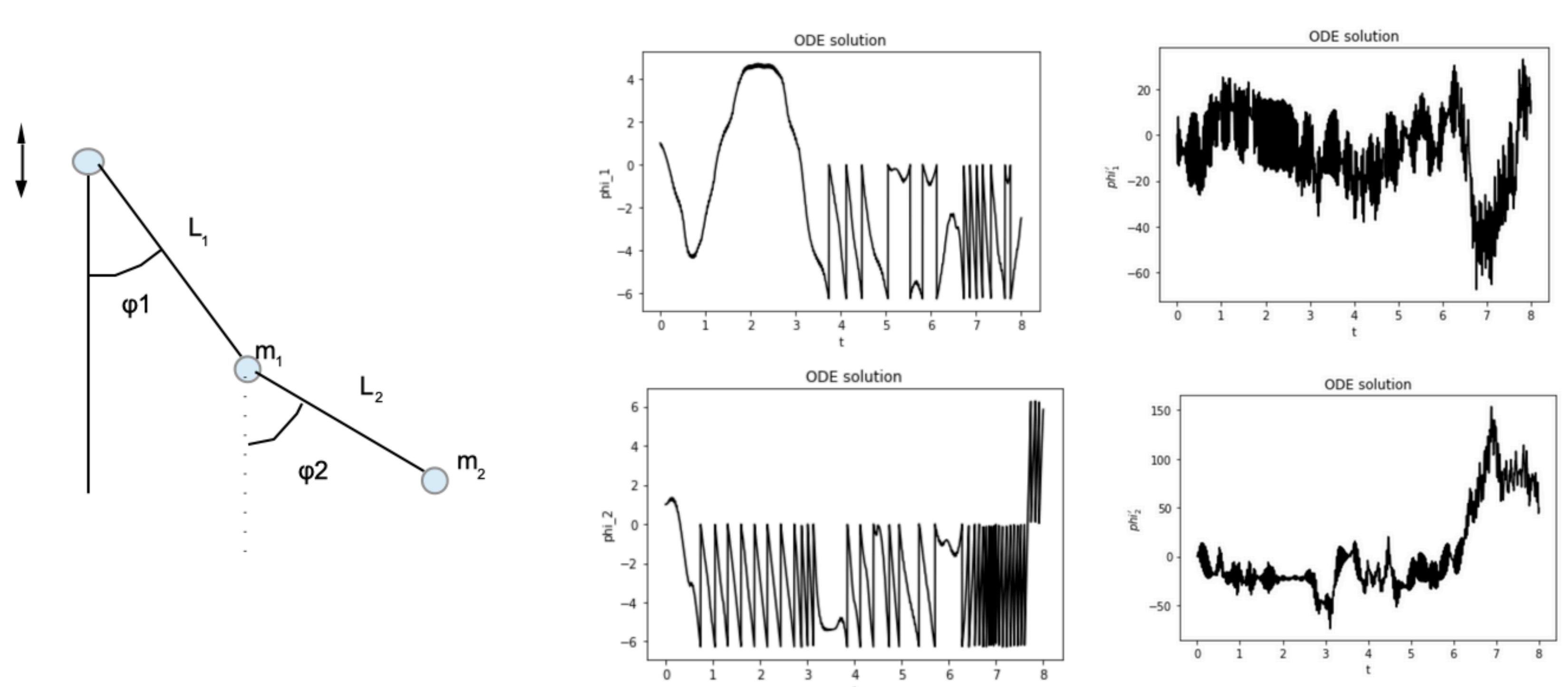
- The Double Inverted Pendulum (DIP) is a nonlinear, unstable, and fast reacting system.
- Various controllers, such as the LQR, can provide stability and control within the system. [1]
- Additionally, new algorithms to correct for the oscillations seen in the system are being explored to more efficiently achieve this stable point. [2]
- Our goal was to not only decompose and understand this DIP system, but also to extract the exact stable points of the system through mathematical modeling, and to prove these points of stability using a Python simulator.

Scientific Challenges

- Adding a secondary arm into the inverse pendulum system increases the overall complexity of the ODEs that govern the dynamics of system.
- Turns into two 2nd order ODE equations of motion that need to be solved simultaneously.

Potential Applications

- This model can provide key insights for certain aerospace and mechatronic systems that rely on keeping the system at a stable node.
- It additionally can reveal a great deal on the estimation strategies applied to mechatronic systems. [3]
- Essentially, a relatively easy-to-build system that can be analyzed and studied for creating control and estimation methodologies for other systems. [1]



Set of Phi versus time plots corresponding to the two arms of the pendulum. Used equal masses (1g) and arm lengths (1m), simulated with a vibrating base with amplitude A=0.05m and Ω = 180 Hz

Team Members:

Kevin Luna (Mentor)
Zachary Kirch
Devin Johnson

Methodology

1. Initially, the Lagrangian equation for the DIP was derived based on the overall Kinetic Energy and Potential Energy of the system.

2. Then the resulting equations of motions were found based on the Lagrangian of the system. There are two equations produced in this instance because there are two angle we are interested in exploring.

$$(m_1+m_2)(sin(\phi_1(t))(a\Omega^2 cos(t\Omega)+g)+l_1\ddot{\phi}_1(t))+l_2m_2\ddot{\phi}_2(t)cos(\phi_1(t)-\phi_2(t))+l_2m_2\dot{\phi}_2^{\,2}(t)sin(\phi_1(t)-\phi_2(t))=0$$

$$sin(\phi_2(t))(a\Omega^2cos(t\Omega)+g)+l_1\ddot{\phi}_1(t)cos(\phi_1(t)-\phi_2(t))-l_1\dot{\phi}_1(t)^2sin(\phi_1(t)-\phi_2(t))+l_2\ddot{\phi}_2(t)=0$$

3. From these equations, a Python script that accepted a 4 dimensional array and produced a corresponding 4 dimensional output was implemented.

4. A self-made RK4 ODE solver was used to produce and effective solution, giving us the data needed to produce 4 plots of the angles and their derivatives with respect to time.

5. An additional piece of code was used to plot the path of the pendulum arm in Cartesian Coordinates.

6. At this point, would have physically modeled this system to produce real world data and compared, but did not have resources to do so.

Results

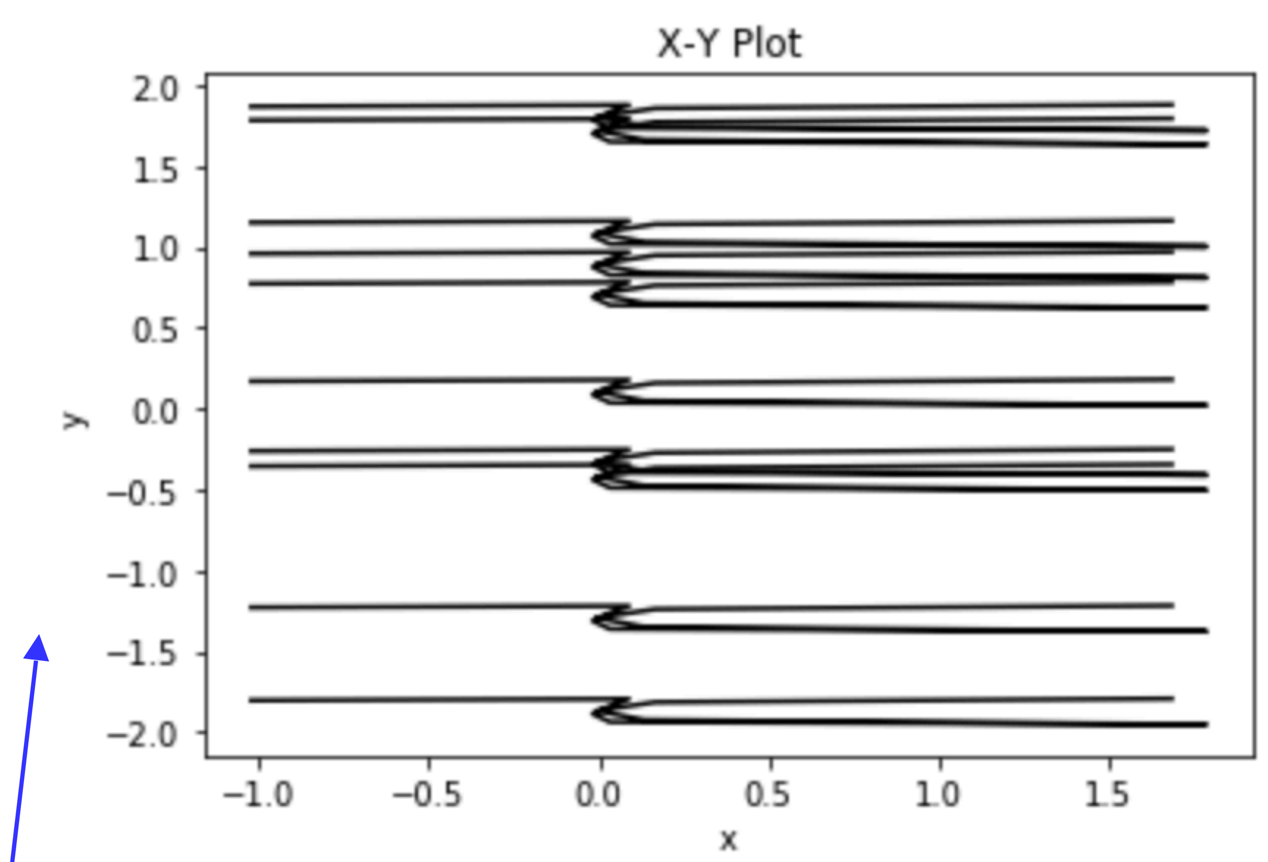
1. The Phi plots shown here express how the phi angles changes with respect to time (top left and bottom left), and the magnitude of their angular speed (top right and bottom right).

2. The Cartesian plot of the effective motion of the outermost ball of the DIP system was also analyzed, and gave some insights as to how the second arm oscillates between points while the first arm has its own oscillatory motion.

3. Further analysis must be done on the system, as the various initial conditions tested did not provide much insight in the Phi plots or the Cartesian plots. We might explore creating two Cartesian plots for both arms

Glossary of Technical Terms

Linear Quadratic Regulator Controller: A controller that utilizes the fact that the system is defined by a set of Linear ODE and whose cost can be modeled by a quadratic function.
Lagrangian Equation: a mechanical equation which relates to the KE and PE of the system, gives stability of system
Rugan Katta (RK) 4 ODE Solver: Method of continual estimations of solutions to ODE, produces accurate numerical solutions



Cartesian Plot of the motion of the end of the 2nd arm of the DIP system

References

1. B. Xu, Y. Lyu, S. Gadsden, "Estimation and Control of a Double-Inverted Pendulum", CSME International Congress, 1-6 (2018)
2. K. Srikanth, G. Kumar, "Stabilization at Upright Equilibrium Position of a Double Inverted Pendulum with Unconstrained BAT optimization", International Journal on Computational Science and Applications, 1-15 (2015)
3. Q. Li, W. Tao, C. Zhang, "Stabilization Control of Double Inverted Pendulum System", IEEE, 1-8 (2008)

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