Practice for Test 3

1) Chapter 10 Check Your Understanding: 1, 3, 5, 7
2) Chapter 10 Review: 9, 11, 15, 28, 33(a,b)
3) Chapter 11 Check Your Understanding: 1, 3, 5, 9, 11
4) Chapter 11 Review: 3, 7, 19, 29, 31, 33

5) State if the following are true or false. Be sure to justify your answers and correct if false.

a) If the power series $\sum_{n=0}^{\infty} b_n (x-4)^n$ has radius of convergence 5 then the series converges for $-5 \leq x \leq 5$.

b) The Taylor series for $\frac{1}{1-x}$ centered at $x = 0$ has radius of convergence equal to infinity.

c) The differential equation

$$\frac{dx}{dt} = \sin(x) + t$$

is separable (i.e. it can be solved using separation of variables).

d) The differential equation

$$\frac{dP}{dt} = -P$$

has a stable equilibrium solution.

e) To solve the differential equation $\frac{dy}{dt} = t^3 + y$ with $y = 2$ when $t = 0$, if one uses Euler's method with $\Delta t = 0.3$, he/she will get the approximations $y(0.3) = 1$ and $y(0.6) = 4$.

6) Write the first four nonzero terms in the Taylor series for the following functions.

a) $\sin(x)$ around $x = 0$. b) $\cos 3x$ around $x = 0$. c) xe^x around $x = 0$.

d) $\frac{1}{x}$ around $x = 1$. e) $\arctan(x)$ around $x = 0$. f) $\ln(-x)$ around $x = -1$.

g) $x^4 + 2x$ around $x = 0$. h) $x^4 + 2x$ around $x = 2$. f) $\frac{1}{(1-x)^3}$ around $x = 0$.

7) Find the following by recognizing the appropriate Taylor series.

a) $1 + (0.1)^2 + (0.1)^3 + (0.1)^4 + (0.1)^5 + \cdots$ b) $\sum_{n=1}^{\infty} \frac{(0.3)^n}{n}$

c) $1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \cdots$ d) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

e) $\sum_{n=1}^{\infty} 2^n \frac{x^n}{n!}$ f) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{2n+3}}{(2n+3)!}$

8) Compute the radius of convergence for the following power series.

a) $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2} (x-3)^n$ b) $\sum_{n=0}^{\infty} \frac{9^n (n+1)^3}{n!} (x+1)^n$ c) $\sum_{n=3}^{\infty} n(-3)^n x^{2n+1}$

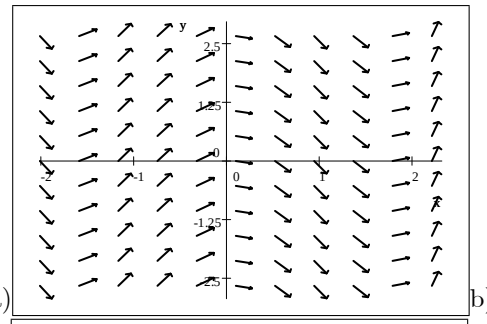
d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3} (x-4)^{2n}$

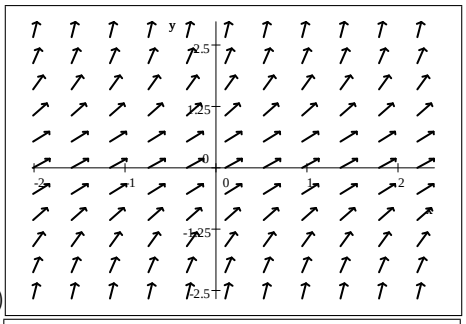
9) a) Use Taylor series to put the following functions in order from smallest to largest for positive values of x near zero: $1 - x + x^2$, $\frac{1}{1+x}$, e^{-x}, $\cos x$, $1 - x$

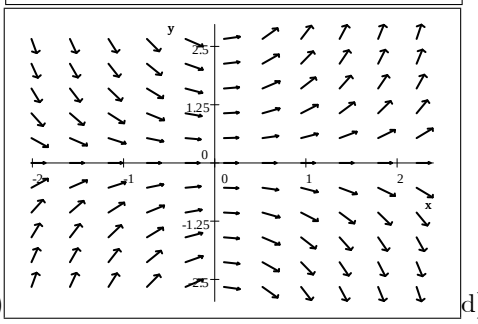
b) Now dow the same for negative values of x near zero.

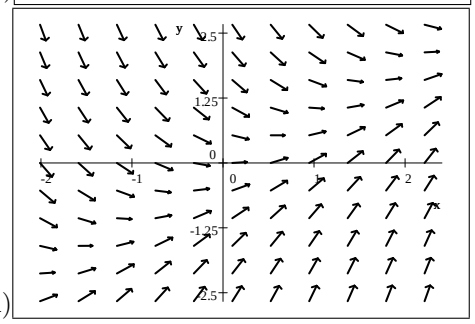
10) Give reasons why each of the following slope fields cannot be the slope field for the differential equation

$$\frac{dy}{dx} = (x-1)\, y$$

a) 

b) 

c) 

d) 

11) Decide which of the following series converge and which diverge. Justify your answer:

$\sum_{n=1}^{\infty} \frac{1}{2^n}$, $\sum_{n=1}^{\infty} \frac{1}{2^2}$, $\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$, $\sum_{n=1}^{\infty} n^{-1}$, $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$, $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$, $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

12) Chapter 9 Check Your Understanding: 9, 11, 13, 17, 19, 21, 27, 29, 37, 39, 41, 45

13) Chapter 9 Review: 23, 25, 27, 29, 35, 43, 47