Review for Test 2

Note: Answers in the back of the book are omitted.

1. Chapter 8 Check Your Understanding: 1, 3, 4, 6, 7, 8

   4. True

   6. False. Only if the density is a function of the distance from one point (usually the city center).

   8. False. I got 4 for the inner city and 12 for the suburbs.

2. Chapter 9 Check Your Understanding: 1, 3, 5, 7, 17, 19, 34

   34. False. Consider harmonic series $1 + 1/2 + 1/3 + 1/4 + \cdots$

3. Chapter 9 Review: 1, 7, 9, 13, 23, 27.

4. Chapter 8 Review: 21, 23, 31, 42(a), 43, 45, 48

   42(a). $2\int_0^1 \left(\sqrt{x} - x^2\right) dx = \frac{4}{3}g$

   48.21, 600 ft-lb

5. a.Compute the volume of a solid whose base is the region bounded by $y = 4 - x^2$ and the x-axis and whose cross-sections perpendicular to the x-axis are equilateral triangles.

   Ans:

$$\frac{\sqrt{3}}{2}\int_{-2}^{2}\left(4 - x^2\right)^2 dx = \frac{256}{15}\sqrt{3} \approx 29.56$$

   b.Now compute the volume of the solid with the same base but whose cross-sections perpendicular to the y-axis are equilateral triangles.

   Ans:

$$2\sqrt{3}\int_{0}^{4}(4 - y)\, dy = 16\sqrt{3} \approx 27.713$$

6. Find the volume of a region whose base is a triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 1)$ and whose cross-section perpendicular to the x-axis are semicircles.

   Ans:

$$\frac{\pi}{8}\int_{0}^{2}\left(\frac{2 - x}{2}\right)^2 dx = \frac{\pi}{12} \approx 0.261\,80$$

7. Section 8.4: 11, 13.

8. Section 8.5: 7, 11, 17.

9. Show that the following integrals converge or diverge (you must actually show the comparison, not just give an idea of why you think it is based on looking at highest powers, etc):

a) $\int_2^\infty \frac{1}{x+1+\sin x} dx$

$$\frac{1}{x+1+\sin x} \geq \frac{1}{x+2}$$

hence

$$\int_2^\infty \frac{1}{x+1+\sin x} dx \geq \int_2^\infty \frac{1}{x+2} dx$$

which diverges, hence this integral diverges.

b) $\int_1^\infty \frac{1}{x^{10}+2x} dx$

on $[1, \infty)$ we see that

$$\frac{1}{x^{10}+2x} \leq \frac{1}{x^{10}}$$

and since $\int_1^\infty \frac{1}{x^{10}} dx$ converge, this integral must converge.

c) $\int_0^2 \frac{1}{x^3+2} dx$

This integral is only improper at the cube root of -2, which is not in the interval. Hence this integral converges.

d) $\int_1^\infty e^{-x^2} dx$

On this interval

$$\begin{aligned} x^2 &\geq x \\ -x^2 &\leq -x \\ e^{-x^2} &\leq e^{-x} \end{aligned}$$

and since $\int_1^\infty e^{-x} dx$ converges, this integral converges.

10. Write an integral that represents the arclength of the following curves:

a) $y = \sin x$ between $x = 0$ and $x = 2\pi$.

$$\int_0^{2\pi} \sqrt{1+\cos^2 x} dx$$

b) $y = x^3 + 1$ between $x = -1$ and $x = 1$.

$$\int_{-1}^1 \sqrt{1+9x^4} dx$$

c) the parametric curve $x = e^{2t}$, $y = \cos t$ for $t \in [0, 3]$.

$$\int_0^3 \sqrt{4e^{4t}+\sin^2 t} dt$$

d) the ellipse $x = 2\cos t$, $y = 4\sin t$ for $t \in [0, 2\pi]$.

$$\int_0^{2\pi} \sqrt{4\sin^2 t + 16\cos^2 t} dt$$

e) $y = \ln x$ between $x = 1$ and $x = 2$.

$$\int_1^2 \sqrt{1 + \frac{1}{x^2}} dx$$

11. Let R be the region bounded by the curve $y = x^2$, the x-axis, and the lines $x = 1$ and $x = 2$. Compute the volumes of the following solids:

a) the solid defined by rotating R about the y-axis.

$$\pi \int_0^4 (16 - y)\, dy = 56\pi$$

b) the solid defined by rotating R about the x-axis.

$$\pi \int_1^2 x^4 dx = \frac{31}{5}\pi$$

c) the solid defined by rotating R about the line $y = 6$.

$$\pi \int_1^2 \left(36 - (6 - x)^2\right) dx = \frac{47}{3}\pi$$

d) the solid whose base is R and whose cross-sections perpendicular to the x-axis are circles.

$$\pi \int_1^2 \left(\frac{1}{2}x^2\right)^2 dx = \frac{31}{20}\pi$$

e) the solid whose base is R and whose cross-sections perpendicular to the y-axis are equilateral triangles.

$$\frac{\sqrt{3}}{2} \int_1^2 \left(x^2\right)^2 dx = \frac{31}{10}\sqrt{3}$$

12. Suppose a chain is hanging over the side of a platform. If the chain is 3 m long and its mass is 6 kg/m, how much work is required to pull the chain up?

Ans: The work is

$$\int_0^3 6\,(9.8)\, y dy = 264.6 \text{ N.}$$

13. How much work does it take to pump out 28 cubic feet of water from the top of a rectangular container 15 feet high with a square base which is 2 feet by 2 feet (so the water is 7 feet high)?

Ans: The work is

$$\int_0^7 (2)(2)(15-h)(62.4)\,dh = 20093 \text{ ft-lb}.$$

14. Decide which of the following series converge and which diverge. Compute if possible. Justify your answer:

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges since it a geometric series with ratio $\frac{1}{2} < 1$. It is equal to

$$\frac{1/2}{1-1/2} = 1.$$

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$\sum_{n=1}^{\infty} \frac{1}{2^2}$ diverges since it is a multiple of $1+1+1+1+\cdots$ or since the limit of its terms is not zero.

$\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$ converges by integral comparison since $\int_1^{\infty} \frac{1}{x^{1.2}}dx$ converges. Cannot compute the value.

$\sum_{n=1}^{\infty} n^{-1}$ diverges (it is the harmonic series, so there it diverges by the integral test).

$\sum_{n=1}^{\infty} \frac{1}{n^{0.2}}$ diverges by integral comparison since $\int_1^{\infty} \frac{1}{x^{0.2}}dx$ diverges.

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by integral comparison since $\int_1^{\infty} \frac{1}{x^2}dx$ converges. Cannot compute the value.

$\sum_{n=1}^{\infty} \frac{1}{n\ln n}$ diverges by integral comparison, since $\int_1^{\infty} \frac{1}{x\ln x}dx = \int_0^{\infty} \frac{1}{u}du$ which diverges.

$\sum_{n=1}^{\infty} \frac{1}{e^n}$ converges since it is a geometric series. It is equal to $\frac{1/e}{1-1/e}$.

15. True/False, if false correct:

a) All geometric series converge.

False. Geometric series (i.e. $\sum_{n=1}^{\infty} ar^n$) whose ratios (r) are less than 1 in absolute value (i.e. $|r| < 1$) converge.

b) For any p, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges.

False. They converge for $p > 1$ by the integral test.

c) The units kg can denote force.

False. kg denote mass, not force.

d) Mass and weight are the same thing.

False. Weight is a force gravity exerts on an object of a given mass.

e) Force and weight use the same units.

True. Weight is the force of gravity on an object, and hence both force and weight have units Newtons (N) or pounds (lb).

f) If $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} f(n)$ converges.

True. This is the integral test.

g) The sequence $1, 1/2, 1/3, 1/4, 1/5, \ldots$ converges.

False. This is the harmonic series and we can see that it diverges by the integral test or by the argument given in class.

h) The series $\sum_{n=2}^{\infty} \frac{n^2-1}{n^2}$ converges.

False. We can do the integral test and see that $\int_2^\infty \frac{x^2-1}{x^2} dx = \int_2^\infty \left(1 - \frac{1}{x^2}\right) dx$ diverges or we see that $\lim_{n\to\infty} \frac{n^2-1}{n^2} = 1 \neq 0$ so the series diverges.

i)

$$\sum_{n=2}^{9} 7\frac{1}{3^n} = \frac{7\left(1 - \frac{1}{3^9}\right)}{1 - \frac{1}{3}}$$

False.

$$\sum_{n=2}^{9} 7\frac{1}{3^n} = \frac{\frac{7}{3^2}\left(1 - \frac{1}{3^8}\right)}{1 - \frac{1}{3}}$$