Practice for Test 3

1) Chapter 8 Check Your Understanding: 19-21, 26-30.

2) Chapter 9 Check Your Understanding: 5, 9, 11, 12, 13, 15, 17, 19, 21, 23, 27, 29, 33, 37, 39, 41, 43, 45

3) Chapter 9 Review: 23, 25, 27, 29, 31, 35, 37, 49, 51, 57

4) Chapter 10 Check Your Understanding: 1, 3, 5, 7

5) Chapter 10 Review: 9, 11, 15, 28, 30, 33, 35, 41

6) State if the following are true or false. Be sure to justify your answers and correct if false.

a) If the power series $\sum_{n=0}^{\infty} b_n (x-4)^n$ has radius of convergence 5 then the series converges for $-5 \leq x \leq 5$.

b) The Taylor series for $\frac{1}{1-x}$ centered at $x = 0$ has radius of convergence equal to infinity.

7) Write the first four nonzero terms in the Taylor series for the following functions.

a) $\sin(x)$ around $x = 0$. b) $\cos 3x$ around $x = 0$. c) xe^x around $x = 0$.

d) $\frac{1}{x}$ around $x = 1$. e) $\arctan(x)$ around $x = 0$. f) $\ln(-x)$ around $x = -1$.

g) $x^4 + 2x$ around $x = 0$. h) $x^4 + 2x$ around $x = 2$. f) $\frac{1}{(1-x)^3}$ around $x = 0$.

8) Find the following by recognizing the appropriate Taylor series.

a) $1 + (0.1)^2 + (0.1)^3 + (0.1)^4 + (0.1)^5 + \cdots$ b) $\sum_{n=1}^{\infty} \frac{(0.3)^n}{n}$

c) $1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \cdots$ d) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

e) $\sum_{n=1}^{\infty} 2^n \frac{x^n}{n!}$ f) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{2n+3}}{(2n+3)!}$

9) Decide which of the following series converge and which diverge. Compute the sum if possible. Justify your answer:

$\sum_{n=1}^{\infty} \frac{1}{2^n}$, $\sum_{n=1}^{\infty} \frac{1}{2^2}$, $\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$, $\sum_{n=1}^{\infty} n^{-1}$, $\sum_{n=1}^{\infty} \frac{1}{n^{0.2}}$, $\sum_{n=1}^{\infty} \frac{1}{n^2}$, $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$, $\sum_{n=1}^{\infty} \frac{1}{e^n}$

10) True/False, if false correct:

a) All geometric series converge.

b) For any p, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges.

c) The density function and the cumulative distribution function are the same.

d) If $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} f(n)$ converges.

e) The sequence $1, 1/2, 1/3, 1/4, 1/5, \ldots$ converges.

f) The series $\sum_{n=2}^{\infty} \frac{n^2-1}{n^2}$ converges.

g)

$$\sum_{n=2}^{9} 7\frac{1}{3^n} = \frac{7\left(1-\frac{1}{3^9}\right)}{1-\frac{1}{3}}$$

11) Compute the radius of convergence for the following power series.
a) $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2}(x-3)^n$ b) $\sum_{n=0}^{\infty} \frac{9^n (n+1)^3}{n!}(x+1)^n$ c) $\sum_{n=3}^{\infty} n(-3)^n x^{2n+1}$
d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3}(x-4)^{2n}$

12) a) Use Taylor series to put the following functions in order from smallest to largest for positive values of x near zero: $1-x+x^2$, $\frac{1}{1+x}$, e^{-x}, $\cos x$, $1-x$
b) Now do the same for negative values of x near zero.

13) a) Write $5-12i$ in the form $R\,e^{i\theta}$.
b) Write $6e^{i(5\pi/4)}$ in the form $a+bi$.
c) Using $\left(e^{i\theta}\right)^2 = e^{2i\theta}$, show that

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta.$$

d) By differentiating $e^{i\theta}$, derive the derivatives of $\cos\theta$ and $\sin\theta$.