Written Homework 8 for Oct. 30, 2013

A) Determine if the following integral converges or diverges. If it converges, give an upper bound. Be sure to justify your answer.

\[
\int_0^{\pi/4} \frac{\cos x}{x^2} \, dx.
\]

B) Jeff works for an international corporation that is based in Cincinnati. He has been assigned to work in Singapore for the year. Singapore has a tax rate of 20% and Jeff’s annual salary is $200,000.

1) Compute the amount of taxes Jeff owes for his salary.

2) The company has agreed to pay all taxes that he owes as part of his benefits. Since it pays for the taxes Jeff owes, his income is increased by the amount you calculated in part 1. How much tax does he owe on that portion.

3) Continuing the process, write an infinite sum that describes the total amount of income that Jeff will receive. Also write an infinite sum that describes the total amount of tax Jeff pays.

4) Explain how the infinite sums are geometric series.

C) Consider the sum

\[
\sum_{n=1}^{\infty} \frac{1}{n^2}.
\]

1) Show that it converges using the integral test.

2) Suppose I wanted to approximate this sum by a finite sum

\[
\sum_{n=1}^{N} \frac{1}{n^2}.
\]

How small can I make \(N\) so that my approximation is correct to 2 decimal places (that is, the error is less than 10^{-2})? Hint: Use an integral to get a bound for \(\sum_{n=N+1}^{\infty} \frac{1}{n^2} \).