Review for Test 1

1. From Chapter 7 Check Your Understanding (p. 389): 1, 3, 5, 6

Answer to 6: No, it involves natural logarithms.

2. From Chapter 7 Review (pp. 387-388): 1, 3, 17, 19, 21, 23, 29, 31, 43, 45, 53, 57, 59, 67, 73, 97, 107, 121, 123, 127, 129, 133, 137, 139. 163, 165, 167

3. Suppose $f(1) = 5, f(0) = -1, \int_0^1 f(t)\,dt = 3$.Then compute:

a. $\int_{-1}^{0} f(x+1)\,dx = \int_0^1 f(u)\,du = 3$ (substitution was $u = x+1$).

b. $\int_{1/2}^{1} f(2x-1)\,dx = \frac{1}{2}\int_0^1 f(w)\,dw = \frac{3}{2}$ (substitution was $w = 2x+1$).

c. $\int_0^1 xf\left(x^2\right)dx = \frac{1}{2}\int_0^1 f(t)\,dt = \frac{3}{2}$ (substitution was $t = x^2$).

d. $\int_0^1 xf'(x)\,dx = xf(x)\,|_0^1 - \int_0^1 f(x)\,dx = 5 - 3 = 2$ (integration by parts)

e. $\int_0^1 \frac{f'(x)}{1+[f(x)]^2}dx = \int_{-1}^{5} \frac{1}{1+s^2}ds = \arctan(5) - \arctan(1) = \arctan(5) - \frac{\pi}{4}$ (substitution was $s = f(x)$)

4. Write

$$\frac{x^5+3x}{x^4-1}$$

in the form

$$p(x) + \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{x+1}$$

where $p(x)$ is a polynomial and A, B, C and D are numbers.

Answer:

$$\begin{aligned}\frac{x^5+3x}{x^4-1} &= x + \frac{4x}{x^4-1}\\ &= x + \frac{-2x}{x^2+1} + \frac{1}{x-1} + \frac{1}{x+1}\end{aligned}$$

5. Show that $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$ for any positive n [hint: write $\sin^n x$ as $\left(\sin^{n-1}x\right)(\sin x)$ and integrate by parts].

Answer:

$$\begin{aligned}\int \sin^n x\,dx &= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\cos^2 x\,dx\\ &= -\sin^{n-1}x\cos x + (n-1)\int \left(\sin^{n-2}x\right)\left(1-\sin^2 x\right)\,dx\\ &= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx - (n-1)\int \sin^n x\,dx\end{aligned}$$

where the integration by parts is

$$\begin{aligned} u &= \sin^{n-1}x & dv &= \sin x\,dx\\ du &= (n-1)\sin^{n-2}x\cos x\,dx & v &= -\cos x.\end{aligned}$$

now solve for $\int \sin^n x \, dx$.

6. Compute $\int \sin(\ln y) \, dy$ using integration by parts twice.