Review for Test 1

1. From Chapter 7 Check Your Understanding (p. 389): 1, 3, 5, 6

Answer to 6: No, it involves natural logarithms.

2. From Chapter 7 Review (pp. 387-388): 1, 3, 17, 19, 21, 23, 29, 31, 43, 45, 53, 57, 59, 67, 73, 97, 107, 121, 123, 127, 129, 133, 137, 139, 163, 165, 167

3. Suppose \( f(1) = 5, f(0) = 1 \); \( R_0^1 f(t) dt = 3 \). Then compute:

   a. \( \int_1^0 f(x) dx = \int_0^1 f(t) dt = 3 \) (substitution was \( u = x + 1 \)).

   b. \( \int_1^{1/2} f(2x - 1) dx = \frac{1}{2} \int_0^1 f(w) dw = \frac{3}{2} \) (substitution was \( w = 2x + 1 \)).

   c. \( \int_0^1 xf(x^2) dx = \frac{1}{2} \int_0^1 f(t) dt = \frac{3}{2} \) (substitution was \( t = x^2 \)).

   d. \( \int_0^1 xf'(x) dx = xf(x) \int_0^1 f(x) dx = 5 - 3 = 2 \) (integration by parts)

   e. \( \int_0^1 \frac{f'(x)}{f(x)} dx = f^{5/2} - \frac{3}{2} \int_0^1 \frac{1}{f(x)} dx = \arctan(5) - \arctan(1) = \arctan(5) - \frac{\pi}{4} \) (substitution was \( s = f(x) \)).

4. Write

\[
\frac{x^5 + 3x}{x^4 - 1}
\]

in the form

\[
p(x) + \frac{Ax + B}{x^2 + 1} + \frac{C}{x - 1} + \frac{D}{x + 1}
\]

where \( p(x) \) is a polynomial and \( A, B, C \) and \( D \) are numbers.

Answer:

\[
\frac{x^5 + 3x}{x^4 - 1} = x + \frac{4x}{x^3 - 1} = x + \frac{-2x}{x^2 + 1} + \frac{1}{x - 1} + \frac{1}{x + 1}
\]

5. Show that \( \int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx \) for any positive \( n \) [hint: write \( \sin^n x \) as \( (\sin^{n-1} x)(\sin x) \) and integrate by parts].

Answer:

\[
\int \sin^n x \, dx = -\sin^{n-1} x \cos x + (n - 1) \int \sin^{n-2} x \cos^2 x \, dx
\]

\[
= -\sin^{n-1} x \cos x + (n - 1) \int (\sin^{n-2} x)(1 - \sin^2 x) \, dx
\]

\[
= -\sin^{n-1} x \cos x + (n - 1) \int \sin^{n-2} x \, dx - (n - 1) \int \sin^n x \, dx
\]

where the integration by parts is

\[
u = \sin^{n-1} x \quad dv = \sin x \, dx
\]

\[
\begin{align*}
    du & = (n - 1) \sin^{n-2} x \cos x \, dx & v & = -\cos x
\end{align*}
\]
now solve for \( \int \sin^n x \, dx \).

6. Compute \( \int \sin (\ln y) \, dy \) using integration by parts twice.