Power series and Taylor series 1

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1. Find the radius of convergence and interval of convergence for the following power series:
   a. Easy:
      \[ \sum_{n=0}^{\infty} \frac{x^n}{n^2} \]
   b. Easy:
      \[ \sum_{n=0}^{\infty} \frac{(x + 3)^n}{n^2} \]
   c. Harder:
      \[ \sum_{n=0}^{\infty} \frac{x^n}{n} \]
   d. Harder:
      \[ \sum_{n=0}^{\infty} \frac{(x + 3)^n}{n!} \]
   e. Hard:
      \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{3^n} \]
   f. Hard:
      \[ \sum_{n=0}^{\infty} (n!) x^n \]

2. Find the Taylor polynomial of degree 5 and the Taylor series for the following functions centered at the given points. Compute the radius of convergence.
   a. Easy: \( e^x \) centered at 0.
   b. Easy: \( 1/(1 + x) \) centered at 0.
   c. Harder: \( x^3 + x^6 \) centered at 2.
   d. Harder: \( \frac{1}{x} \) centered at 1.
   e. Harder: \( \frac{1}{x} \) centered at -1.
   f. Hard: \( \sin x \) centered at 0.
   g. Hard: \( \cos x \) centered at 0.
   h. Hard: \( \ln(1 + x) \) centered at 0.
   i. Hard: \( \arctan (x) \) centered at 0.