Math 322 Section 3 Written Homework 3

1) The delta function \( \delta(x) \) is defined to be the function such that for any function \( f(x) \),

\[
\int_a^b \delta(x) f(x) \, dx = \begin{cases} 
  f(0) & \text{if } a < 0 < b \\
  0 & \text{if } a \geq 0 \text{ or } b \leq 0
\end{cases}
\]

Actually, it can be shown that no such function exists, but it still has a Fourier series since, for instance,

\[
\int_{-\pi}^{\pi} \delta(x) \cos 3x \, dx = \cos(0) = 1.
\]

1) Calculate \( \int_a^b \delta(x - \frac{\pi}{2}) f(x) \, dx \) for any function \( f(x) \).

2) Write the Fourier series for the delta function \( \delta(x - \frac{\pi}{2}) \) gotten as an odd periodic half range expansion on the interval \([0, \pi]\).

3) Show that the series does not converge for \( x = \frac{\pi}{2} \).

4) Use separation of variables to find the solution to the heat equation

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \]

where \( u(x, t) \) satisfies \( u(0, t) = u(\pi, t) = 0 \) and \( u(x, 0) = \delta(x - \frac{\pi}{2}) \).

5) Explain why we say that the heat equation shrinks high frequency modes faster than low frequency modes.