Ordered Field Axioms for set S with operations + and •, and relation <

A1. For all $x, y \in S$, $x+y \in S$ and if $x = w$ and $y = z$ then $x+y = w+z$.
A2. For all $x, y \in S$, $x + y = y + x$.
A3. For all $x, y, z \in S$, $x + (y + z) = (x + y) + z$
A4. There exists $0 \in S$ such that $x + 0 = x$ for all $x \in S$.
A5. For each $x \in S$, there exists a unique $-x \in S$ such that $x+(-x) = 0$.
M1. For all $x, y \in S$, $xy \in S$ and if $x = w$ and $y = z$ then $xy = wz$.
M2. For all $x, y \in S$, $xy = yx$.
M3. For all $x, y, z \in S$, $x(yz) = (xy)z$
M4. There exists $1 \in S$ such that $1 \neq 0$ and $x \cdot 1 = x$ for all $x \in S$.
M5. For each $x \in S \setminus \{0\}$, there exists a unique $\frac{1}{x} \in S$ such that $x \cdot \frac{1}{x} = 1$.
DL. For all $x, y, z \in S$, $x(y + z) = xy + xz$.
O1. For all $x, y \in S$, exactly one of $x < y$, $x = 0$, or $y < x$ holds.
O2. For all $x, y, z \in S$, if $x < y$ and $y < z$ then $x < z$.
O3. For all $x, y, z \in S$, if $x < y$ then $x + z < y + z$.
O4. For all $x, y, z \in S$, if $x < y$ and $z > 0$ then $xz < yz$.

Theorem: Let $x, y, z \in S$. Then:

a. If $x + z = y + z$ then $x = y$.
b. $x \cdot 0 = 0$.
c. $(-1)x = -x$.
d. $xy = 0$ if and only if $x = 0$ or $y = 0$.
e. $x < y$ if and only if $-y < -x$.
f. If $x < y$ and $z < 0$ then $yz < xz$.