Math 323: Homework 12 Solutions

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10.23)

Proposition 1 *For all integers $x \geq 8$, x can be written in the form $3m+5n$ where m and n are nonnegative integers.*

Proof. We induct on x. Notice that $8 = 3(1) + 5(1)$ and so the basis is proven. Suppose $x \geq 8$ can be written $x = 3m + 5n$. If $n \neq 0$, then we can write

$$\begin{aligned} x + 1 &= 3m + 5n + 6 - 5 \\ &= 3(m+2) + 5(n-1) \end{aligned}$$

and $m + 2$ and $n - 1$ are nonnegative integers. If $n = 0$, then since $x = 3m \geq 8$, we must have $m \geq 3$. We can now write

$$x + 1 = 3m - 9 + 10 = 3(m-3) + 5(2)$$

with $m - 3 \geq 0$. ■

11.3b) Note that there are a number of other ways to do this problem.

Proposition 2 $(-x)\,y = -(xy)$ *and* $(-x)(-y) = xy$.

Proof. For the first statement, we can try

$$\begin{aligned} xy + (-x)\,y &= (x + -x)\,y \\ &= 0\,(y) \\ &= 0, \end{aligned}$$

where we have used DL in the first equality, A4 in the second equality, and Theorem part b in the third. Thus $(-x)\,y = xy$.

For the second, we can show that

$$(-x)(-y) = -(x(-y))$$

using the first part. Similarly,

$$x(-y) = (-y)\,x = -yx = -xy$$

using the first part and M2. We see that

$$-xy + (-x)(-y) = 0 = -xy + xy$$

using also A4, so$(-x)(-y) = xy$ using the Theorem part a. ■

11.3d)

Proposition 3 *If $xz = yz$ and $z \neq 0$ then $x = y$.*

Proof. Since $z \neq 0$, there exists $\frac{1}{z}$ by M5. Multiplying the original equality using M1, we have

$$\begin{aligned} (xz)\left(\frac{1}{z}\right) &= yz\left(\frac{1}{z}\right) \\ x \cdot 1 &= z \cdot 1 \\ x &= z \end{aligned}$$

where we have used associativity (M3) and M4/M5. ■