Math 323: Homework 1 Solutions

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1.3b) The interval $[0,3]$ is finite. Its negation is "The interval $[0,3]$ is infinite."

1.3e) If $x > 3$ then $f(x) > 7$. Its negation is "$x > 3$ and $f(x) \leq 7$."

1.3f) If f is continuous and A is connected, then $f(A)$ is connected. Its negation is "f is continuous and A is connected and $f(A)$ is not connected."

1.3g) If K is compact, then K is closed and bounded. *Its* negation is "K is compact and K is either not closed or not bounded."

1.4d) $x < 5$ or $x > 7$. Its negation is "$5 \leq x \leq 7$." (Note: this is a shortened form of" $x \geq 5$ and $x \leq 6$.")

1.5a) "M has a zero eigenvalue" is the antecedent and "M is singular" is the consequent.

1.5c) "The sequence is Cauchy" is the antecedent and "the sequence is bounded" is the consequent.

1.7b) We show the truth table for $[p \wedge (p \implies q)] \implies q$.

p	q	$[p$	$\wedge$	$(p \implies q)]$	$\implies$	q
T	T		T	T	T	
T	F		F	F	T	
F	T		F	T	T	
F	F		F	T	T	

The statement is a tautology.

1.7c) We show the truth table for $[p \implies (q \wedge \sim q)] \implies \sim p$.

p	q	$[p$	$\implies$	$(q \wedge \sim q)]$	$\iff$	$\sim p$
T	T		F	F	T	F
T	F		F	F	T	F
F	T		T	F	T	T
F	F		T	F	T	T

The statement is a tautology.

1.9a) Claim: $2 \leq 3$ and 7 is prime. This statement is true because both of "$2 \leq 3$" and "7 is prime" are true.

1.9b) Claim: $6 + 2 = 8$ or 6 is prime. This statement is true because $6 + 2 = 8$.

1.9d) Claim: If 3 is prime, then $3^2 = 9$. This statement is true because 3 is prime and $3^2 = 9$.

1.9e) Claim: If 3 is not prime, then $3^2 \neq 9$. This statement is true because 3 is prime.

1.9f) Claim: If $3^2 = 9$ then 3 is not prime. This statement is false because the antecedent ($3^2 = 9$) is true but the consequent (3 is not prime is false).

1.9i) Claim: if both $2 + 5 = 7$ and $2 \cdot 5 = 7$, then $2^2 + 5^2 = 7^2$. This is true because $2 \cdot 5 \neq 7$ gives that the antecedent is false, hence the implication is true.

1.9j) Claim: it is not the case that $2 + 3 \neq 5$. This is equivalent to saying $2 + 3 = 5$, which is true.