Math 323: Homework 2 Solutions

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January 28, 2013

1.11) Suppose p is the statement "Misty is a dog," and q is the statement "Misty is a cat." The following are expressed in symbols:

a) Misty is not a cat, but she is a dog. $\sim q \wedge p$.

b) Misty is a dog or a cat, but not both. $(p \vee q) \wedge (\sim (p \wedge q))$ or $(p \vee q) \wedge (\sim p \vee \sim q)$.

c) Misty is a dog or a cat, but she is not a cat. $(p \vee q) \wedge \sim q$.

d) If Misty is not a dog, then Misty is a cat. $\sim p \Longrightarrow q$.

e) Misty is a dog iff she is not a cat. $p \Longleftrightarrow \sim q$.

1.14a)

p	q	$p \wedge q$	$\Longleftrightarrow$	$q \wedge p$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

1.14d)

p	q	r	$[p$	$\vee$	$(q$	$\vee$	$r)]$	$\Longleftrightarrow$	$[(p$	$\vee$	$q)$	$\vee$	$r]$
T	T	T		T		T		T		T		T	
T	T	F		T		T		T		T		T	
T	F	T		T		T		T		T		T	
T	F	F		T		F		T		T		T	
F	T	T		T		T		T		T		T	
F	T	F		T		T		T		T		T	
F	F	T		T		T		T		F		T	
F	F	F		F		F		T		F		F	

1.14f)

p	q	r	$[p$	$\vee$	$(q$	$\wedge$	$r)]$	$\Longleftrightarrow$	$[(p$	$\vee$	$q)$	$\wedge$	$(p$	$\vee$	$r)]$
T	T	T		T		T		T		T		T		T	
T	T	F		T		F		T		T		T		T	
T	F	T		T		F		T		T		T		T	
T	F	F		T		F		T		T		T		T	
F	T	T		T		T		T		T		T		T	
F	T	F		F		F		T		T		F		F	
F	F	T		F		F		T		F		F		T	
F	F	F		F		F		T		F		F		F	

2.3c)The negation of "no even integer is prime" is "there exists an even integer that is prime."

2.3d) The negation of "$\exists x < 3 \ni x^2 \geq 10$" is "$\forall x < 3,\ x^2 < 10$."

2.3e) The negation of "$\forall x$ in $A, \exists y < k \ni 0 < f(y) < f(x)$" is "$\exists x$ in $A \ni \forall y < k,\ 0 \leq f(y)$ or $f(y) \geq f(x)$."

2.3f) The negation of "If $n > N$, then $\forall x$ in S, $|f_n(x) - f(x)| < \varepsilon$" is "$n > N$ and $\exists x$ in $S \ni |f_n(x) - f(x)| \geq \varepsilon$," if both n and N are defined somewhere else as particular numbers. Usually in this sort of statement, N is defined somewhere else and it should be true for all n, in which case the statement is really "for all $n > N$, $\forall x$ in S, $|f_n(x) - f(x)| < \varepsilon$" and the negation is "$\exists n > N$ such that $\exists x$ in $S \ni |f_n(x) - f(x)| \geq \varepsilon$."

2.4a) The negation of "Some basketball players at Central High are short" is "All basketball players at Central High are tall."

2.4e) The negation of "$\forall x \ni 0 < x < 1$, $f(x) < 2$ or $f(x) > 5$" is "$\exists x \ni 0 < x < 1$ and $2 \leq f(x) \leq 5$."

2.4f) The negation of "If $x > 5$, then $\exists y > 0 \ni x^2 > 25 + y$" is "$x > 5$ and $\forall y > 0$, $x^2 \leq 25 + y$" if x has been defined elsewhere. Alternatively, the statement really means "For all $x > 5$, $\exists y > 0 \ni x^2 > 25 + y$" and its negation is "There exists $x > 5$ such that $\forall y > 0$, $x^2 \leq 25 + y$."

2.8) Which of the following best identifies f as a constant function, where x and y are real numbers:
a) $\exists x \ni \forall y$, $f(x) = y$. No, this cannot happen for any function.
b) $\forall x\ \exists y \ni f(x) = y$. No, this is true for any function whose domain is all real numbers.
c) $\exists y \ni \forall x$, $f(x) = y$. Yes, this identifies a constant function.
d) $\forall y\ \exists x \ni f(x) = y$. No, this says a function is onto (or surjective). Certainly it is not true for a constant function.

2.10a) "$\exists x$ in $[3,5] \ni x < 7$" is true. Take $x = 4$, for instance.

2.10b) "$\forall x$ in $[3,5]$, $x \geq 4$" is false. Take $x = 3$, for instance.

2.10c) "$\exists x \ni x^2 \neq 3$" is true. Take $x = 1$, for instance.

2.10d) "$\forall x, x^2 \neq 3$" is false. Take $x = \sqrt{3}$, for instance.

2.10e) "$\exists x \ni x^2 = -5$" is false, since $x^2 \geq 0$ for all real numbers x.

2.10f) "$\forall x$, $x^2 = -5$" is false. In fact, by 2.10e there is not a single value of x for which $x^2 = -5$.

2.10g) "$\exists x \ni x - x = 0$" is true. Take, for instance, $x = 0$.

2.10h) "$\forall x$, $x - x = 0$" is true. This is a basic fact of arithmetic.

2.15) A function $f : A \to B$ is injective iff for every x and y in A, if $f(x) = f(y)$, then $x = y$. The defining condition is:

$$\forall x, y \in A,\ f(x) = f(y) \Rightarrow x = y.$$

The negation is:

$$\exists x, y \in A \ \ni\ f(x) = f(y) \wedge x \neq y.$$

2.17) A function $f : D \to R$ is continuous at $c \in D$ iff for every $\varepsilon > 0$ there is a $\delta > 0$ such that $|f(x) - f(c)| < \varepsilon$ whenever x is in D and $|x - c| < \delta$. The defining condition is

$$\forall \varepsilon > 0,\ \exists \delta > 0 \ni \forall x \in D,\ \ |x - c| < \delta \Rightarrow |f(x) - f(c)| < \varepsilon$$

or

$$\forall \varepsilon > 0,\ \exists \delta > 0 \ni [(x \in D) \wedge |x - c| < \delta] \Rightarrow |f(x) - f(c)| < \varepsilon.$$

Note that in the latter formulation, it is understood that the implication is true "for all x." Its negation is

$$\exists \varepsilon > 0 \ni \forall \delta > 0, \exists x \in D \ \ni [|x - c| < \delta \wedge |f(x) - f(c)| \geq \varepsilon].$$

Note that we need to have "there exists x," otherwise we assume it means "for all x" which is not what we want! This is the case even if we consider the second statement, since the original statement certainly implies the statement is true for all x.