Math 323: Homework 3 Solutions

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2.18) A function is uniformly continuous on a set S iff for every $\varepsilon > 0$ there is a $\delta > 0$ such that $|f(x) - f(y)| < \varepsilon$ whenever x and y are in S and $|x - y| < \delta$.

Defining condition: $\forall \varepsilon > 0\ \exists \delta > 0 \ni \forall x, y \in S$, $|x - y| < \delta \Longrightarrow |f(x) - f(y)| < \varepsilon$.

Negation: $\exists \varepsilon > 0 \ni \forall \delta > 0$, $\exists x, y \in S \ni |\ x - y| < \delta \wedge\ |f(x) - f(y)| \geq \varepsilon$.

2.19) The real number L is the limit of the function $f : D \to R$ at the point c iff for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $x \in D$ and $0 < |x - c| < \delta$.

Defining condition: $\forall \varepsilon > 0\ \exists \delta > 0 \ni \forall x \in D$, $0 < |x - c| < \delta \Longrightarrow |f(x) - L| < \varepsilon$.

Negation: $\exists \varepsilon > 0 \ni \forall \delta > 0$, $\exists x \in D \ni 0 < |x - c| < \delta \wedge |f(x) - L| \geq \varepsilon$.

3.3a) The contrapositive of "If all roses are red, then all violets are blue" is "If there exists a violet that is not blue then there exists a rose that is not red."

3.3b) The contrapositive of "H is normal if H is not regular," which is equivalent to "H is not regular implies H is normal" is "H is not normal implies that H is regular" or "if H is not normal, then H is regular" or "H is regular if H is not normal."

3.3c) The contrapositive of "if K is closed and bounded, then K is compact" is "if K is not compact, then K is not closed or K is not bounded."

3.4a) The converse of "If all roses are red, then all violets are blue" is "If all violets are blue then all roses are red."

3.4b) The converse of "H is normal if H is not regular," which is equivalent to "H is not regular implies H is normal" is "H is not regular implies that H is normal" or "if H is not regular, then H is normal" or "H is not regular if H is normal."

3.4c) The converse of "if K is closed and bounded, then K is compact" is "if K is compact, then K is closed and bounded."

3.6f) For every positive integer n, $3n$ is divisible by 6. A counterexample is $n = 1$, since $3(1) = 3$ is not divisible by 6.

3.6h) Every real number has a reciprocal. A counterexample is the number zero, which has no reciprocal.

3.7a)

Proposition 1 *If p is odd and q is odd, then $p + q$ is even.*

Proof. If p is odd and q is odd, then there are integers k and ℓ such that $p = 2k + 1$ and $q = 2\ell + 1$. Then

$$p + q = 2k + 1 + 2\ell + 1 = 2(k + \ell + 1).$$

Thus $p + q$ is even. ■

3.7b)

Proposition 2 *If p is odd and q is odd, then pq is odd.*

Proof. If p and q are odd, then there exist integers k and ℓ such that $p = 2k + 1$ and $q = 2\ell + 1$. Then we compute

$$\begin{aligned} pq &= (2k+1)(2\ell+1) \\ &= 2(2k\ell + k + \ell) + 1. \end{aligned}$$

Since $2(2k\ell + k + \ell)$ is an integer, pq is odd.. ■

3.8)

Proposition 3 *Let f be the function $f(x) = 3x - 5$. If $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$.*

Proof. We prove the contrapositive. Suppose $f(x_1) = f(x_2)$. Then

$$\begin{aligned} 3x_1 - 5 &= 3x_2 - 5 \\ 3x_1 &= 3x_2 \\ x_1 &= x_2. \end{aligned}$$

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