

Consider the following relation on $[-\pi, \pi]$:

$$x \sim y \text{ iff } x = y \text{ or } x, y \in \{-\pi, \pi\}.$$

- a) Show that this is an equivalence relation.
- b) describe the equivalence classes.
- c) Show that

$$f([x]) = (\cos x, \sin x)$$

is a well-defined function $A \rightarrow \mathbb{R}^2$, where A is the set of equivalence classes of \sim .

- d) Show that if we set the codomain to be

$$B = \{(\cos t, \sin t) \in \mathbb{R}^2 : t \in \mathbb{R}\},$$

then $f : A \rightarrow B$ is a bijection. Hint: you can use inverse trig functions, but be careful of where they exist and what their domains and ranges are!