Let $A$ be the following set:

$$A = \{a, b, c, d, e, f\}.$$ 

Suppose $R$ is an equivalence relation on $A$, and let $E_a$ denote the equivalence class of $a$, $E_b$ denote the equivalence class of $b$, etc. For each of the following, explain why the statement is true (using properties of equivalence relations).

a) We must have $bRb$.

b) If $aRd$ and $dRf$ then the equivalence class of $a$ contains both $d$ and $f$.

c) If $a$ is in the equivalence class of $b$ then $E_b = E_a$.

a) This is the reflexive property.

b) Since $aRd$, we have that $d \in E_a$. Using transitivity, since $aRd$ and $dRf$, we have $aRf$, so $f \in E_d$.

c) If $a \in E_b$, then $aRb$ and, by transitivity, if $cRa$, then $cRb$ and so $E_c \subseteq E_b$. Similarly, since $aRb$, symmetry says that $bRa$, and by the same transitivity argument, $E_b \subseteq E_a$. 