Extra problem: Let J be the index set defined by $J = \{x \in \mathbb{R} : 2 < x\}$.
a. Show

$$\bigcap_{x \in J} \left[-\frac{1}{x}, \frac{1}{x}\right] = \{0\}.$$

Answer: We note that for any $x \in J$, $0 \in \left[-\frac{1}{x}, \frac{1}{x}\right]$, so $\{0\} \subseteq \bigcap_{x \in J} \left[-\frac{1}{x}, \frac{1}{x}\right]$. Now consider $y \neq 0$. Then we can take $x = 2 + \left|\frac{1}{y}\right|$, and then

$$\left|\frac{1}{y}\right| = x - 2 < x$$

and so

$$|y| > \frac{1}{x}$$

and so

$$y \notin \left[-\frac{1}{x}, \frac{1}{x}\right].$$

Thus we have proved that $\bigcap_{x \in J} \left[-\frac{1}{x}, \frac{1}{x}\right] \subseteq \{0\}$ (technically, we proved the contrapositive).
b. Show

$$\bigcup_{x \in J} \left[-\frac{1}{x}, \frac{1}{x}\right] = \left(-\frac{1}{2}, \frac{1}{2}\right).$$

Answer: Suppose $y \in \left(-\frac{1}{2}, \frac{1}{2}\right)$. Then $\frac{1}{|y|} > 2$, and so $x = \frac{1}{|y|} \in J$ and $y \in \left[-\frac{1}{x}, \frac{1}{x}\right]$. Now suppose $y \in \bigcup_{x \in J} \left[-\frac{1}{x}, \frac{1}{x}\right]$, so there exists $x \in J$ such that $y \in \left[-\frac{1}{x}, \frac{1}{x}\right]$. Since $x > 2$, we have that $-\frac{1}{2} < -\frac{1}{x} \leq y \leq \frac{1}{x} < \frac{1}{2}$, and so $y \in \left(-\frac{1}{2}, \frac{1}{2}\right)$.