

Some things to be sure to know for Test 3 (note: this is not ALL you need for the test, but this is a good start)

1) Definitions of function, injective/injection, surjective/surjection, bijective/bijection, domain, codomain, range, image, preimage.

2) Definitions of equinumerous, finite, infinite, denumerable, countable, uncountable

3) Definitions of relation, reflexive, symmetric, transitive, equivalence relation, equivalence class.

4) If  $f : A \rightarrow B$  and  $C, C' \subseteq A$  and  $D, D' \subseteq B$ , then you should be able to prove basic things like:

$$\begin{aligned} C &\subseteq f^{-1}(f(C)) \\ f(f^{-1}(D)) &\subseteq D \\ f(C \cap C') &\subseteq f(C) \cap f(C') \\ f(C \cup C') &= f(C) \cup f(C') \\ f^{-1}(D \cap D') &= f^{-1}(D) \cap f^{-1}(D') \\ f^{-1}(D \cap D') &= f^{-1}(D) \cap f^{-1}(D') \end{aligned}$$

(Recall that if there is an equal, there are two directions to prove. It is extremely important that you know how to start and end these proofs.

5) Basics of compositions of functions: Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions. You should be able to prove:

- a) If  $f$  and  $g$  are surjective, then  $g \circ f$  is surjective.
- b) If  $g \circ f$  is surjective, then  $g$  is surjective, but  $f$  may not be.
- c) If  $f$  and  $g$  are injective, then  $g \circ f$  is injective.
- d) If  $g \circ f$  is injective, then  $f$  is injective, but  $g$  may not be.

6) More things you should be able to prove:

- a) Any finite set is not equinumerous to  $\mathbb{N}$ .
- b)  $\mathbb{N}$  is equinumerous to  $\mathbb{N} \setminus \{1\}$ .
- c)  $\mathbb{N}$  is equinumerous to  $\mathbb{Z}$ .

7) You should know the definition of arbitrary intersection and arbitrary unions. Also, you should be able to prove:

- a)  $\bigcap_{n \in \mathbb{N}} [0, \frac{1}{n}] = \{0\}$
- b)  $\bigcap_{n \in \mathbb{N}} (0, \frac{1}{n}] = \emptyset$
- c)  $\bigcup_{n \in \mathbb{N}} [0, 1 - \frac{1}{n}] = [0, 1)$
- d)  $\bigcup_{x \in (0,1)} [x, \frac{1}{x}] = (0, \infty)$