Chapter Check for Chapter 1

September 16, 2015

1. Let $\mathcal{F}(X, F)$ denote the set of functions from the set $X$ to the field $F$.

   a. Describe the usual vector space structure for $\mathcal{F}(X, F)$ (you do not have to show it is a vector space).

   b. Let $x_0 \in X$ and for each $a \in F$, consider the set $S_a = \{ f \in \mathcal{F}(X, F) : f(x_0) = a \}$. For which values of $a$ is $S_a$ a subspace? Justify your answer.

   c. If $X$ is a finite set, show that the dimension of $\mathcal{F}(X, F)$ is equal to the number of elements in $X$.

2. 

   a. Find a basis for $\mathbb{R}^3$ that contains the vector $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Justify your answer.

   b. Is the set $\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} \right\}$ a basis for $\mathbb{R}^3$? Why or why not?

3. (Comprehensive/graduate option only) Consider the subspace $W = \{(t, t, t) \in \mathbb{R}^3 : t \in \mathbb{R} \}$ of $\mathbb{R}^3$. Give an explicit basis for $\mathbb{R}^3/W$. Justify your answer.