Homework problem from Chapter 5

Let \( A \) be a square matrix with real entries. We can define a matrix \( \exp A \) by the “power series”

\[
\exp A = I + A + \frac{1}{2} A^2 + \frac{1}{3!} A^3 + \cdots
\]

Suppose \( A \) is diagonalizable with eigenvalues \( \lambda_1, \ldots, \lambda_n \) (not necessarily distinct).

a. Give a general formula for \( A^n \) of the form \( Q^{-1} D_n Q \) where \( D_n \) is diagonal. Be sure to specify what \( Q \) and \( D_n \) are precisely.

b. Give a general formula for \( \exp A \) of the form \( Q^{-1} DQ \) where \( D \) is diagonal.

c. Calculate \( \exp \begin{pmatrix} 3 & 12 \\ 0 & -1 \end{pmatrix} \).