MATH 443/543

$\mathcal{TEST}$ 2

December 3^{rd}, 2008

Your Name: ____

Directions:

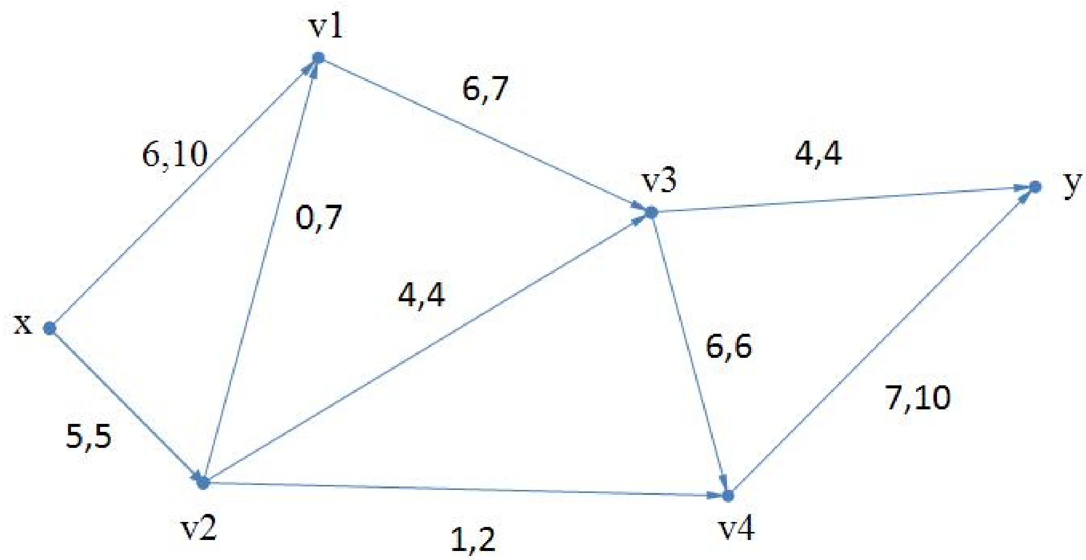
a. You may NOT use your book or your notes or a calculator.
b. Please ask for extra scrap paper if needed.
c. Show all work. Unless otherwise noted, a solution without work is worth nothing.
d. Circle your answers.
e. Good Luck!

Score:

1.	____________
2.	____________
3.	____________
4.	____________
5.	____________
Total	____________

1. (30pts) In the following network, edges are labeled x, y, where x is flow at the edge and y is the capacity of the edge. You are given an initial flow from x to y in the picture.

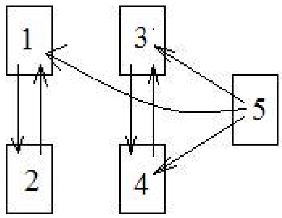
a. (20pts) Use the labelling method to find the max flow. What is its value?



b. (10pts) For which of the following choices of supplies $\sigma(x)$ and demands $\delta(y)$ on the network above, is there a feasible flow? Circle all that apply. (Note: if you cannot find the max flow above, assume that the flow above is maximal, and indicate that you are choosing this.)

$\sigma(x) = 10,\ \delta(y) = 11$
$\sigma(x) = 15,\ \delta(y) = 11$
$\sigma(x) = 11,\ \delta(y) = 15$
$\sigma(x) = 15,\ \delta(y) = 5$

2. (20pts)
Consider the web shown below.



a. (5pts) Give the link matrix A for the web.

b. (10pts) Show that the link matrix A has $\dim V_1(A) > 1$.

c. (5pts) Why does this not contradict the fact that the link matrix A of a strongly connected web has $\dim V_1(A) = 1$? Justify your answer.

3. (15pts) Let F_1 be a graph of order m and F_2 be a graph of order n. Explain why $r\left(F_1, F_2\right) \leq r\left(K_m, K_n\right)$.

4. (20pts) Suppose we have n boys, denoted $B_1, B_2, \ldots, B_n$, and n girls, denoted $G_1, G_2, \ldots, G_n$. Furthermore, suppose each girl has the exact same preference list, preferring B_1 first, then B_2, etc., so that each girl likes B_n the least. Also suppose that each boy has the exact same preference list, preferring G_1, then G_2, then G_3, etc.

a. (5pts) Show that B_1 must be matched to G_1.

b. (10pts) Use induction to show that B_n must be matched to G_n.

c. (5pts) How many days would it take the mating ritual to find this matching?

5. (15pts)

Recall that the page rank was computed by looking at the eigenvector with eigenvalue one of the matrix $M = (1 - m) A + mS$, where A is the link matrix, S is the matrix of all $1/n$ (where n is the number of pages in the web), and m is a number between zero and 1. For this problem, you may assume that A is column stochastic.

Suppose the publisher of the rankings wants to boost the rankings of some of the pages (say, because those pages pay the publisher a fee). He can do this by changing the matrix S. For instance, suppose the web consists of 10 pages and the publisher wants to boost the ranking of the pages numbered 1 and 2. He can adjust rows 1 and 2 of the matrix S to boost the rankings of pages 1 and 2.

What should he do to the matrix S to boost those rankings while maintaining the fact that the rankings are well defined (i.e., there is a unique ranking corresponding to the matrix M with the altered S matrix)? You do not need to give a complete proof, but indicate why you think the rankings will be boosted and why the rankings are still well-defined.