Integer Programming Formulations for Minimum Spanning Forest Problem

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Outline

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1. Introduction
2. Minimum Spanning Tree IP Formulations
3. Minimum Spanning Forest IP Formulations
4. Conclusion

Goals for this talk

- Introduce *mathematical programming* as a general framework to solve decision making problems

- Introduce mathematical programming formulations for *minimum spanning tree* and *minimum spanning forest* problems

Operations Research: science of decision making, science of better

- Some of the mathematical tools to approach decision making?
  - Mathematical Programming
  - Control Theory
  - Decision Analysis
  - Game Theory
  - Queuing Theory
  - Simulation

Mathematical Programming

Definition

A general mathematical program has the form

$$\min_x f(x)$$
$$\text{s.t} \quad x \in X$$

where x is the vector of decision variables, $f(x)$ is the objective function, X is the constraint set, $\{x \in X\}$ is the feasible region.

Different assumptions on $f(x)$ and X results in different classes of mathematical programs

- Linear Programming (LP): $f(x) = cx$, $X = \{Ax \geq b\}$, $x \in \mathbb{R}^n$, $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$.
- Nonlinear Programming (NLP): $f(x)$ nonlinear in x, and/or X a nonlinear set.
- Integer Linear Programming (ILP): Same assumptions as LP, except $x \in \mathbb{Z}^n$
- Mixed Integer LP (MILP): Same assumptions as LP, except $x \in \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2}$

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Recall: Minimum Spanning Tree

- Given a network (G, ϕ), we can define the weight of a subgraph $H \subset G$ as

$$\phi(H) = \sum_{e \in E(H)} \phi(e).$$

Definition

In a *connected* graph G, a minimal spanning tree T is a tree with minimum value.

- MST problem in mathematical programming form:

$$\min_{T} H(T) = \sum_{e \in E(T)} \phi(e)$$

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Minimum Spanning Tree: Subtour Elimination Formulation

- Let $x_{ij} = \begin{cases} 1 & \text{if edge}(i,j) \text{ is in tree} \\ 0 & \text{otherwise} \end{cases}$
- Let x denote the vector formed by x_{ij}'s for all $(i,j) \in E$.
- The MST found by optimal x^*, denoted T^*, will be a subgraph $T^* = (V, E^*)$, where $E^* = \{(i,j) \in E : x^*_{ij} = 1\}$ denotes the selected edge into the spanning tree.
- Subtour elimination formulation is based on the fact that T has no simple cycles and has $n-1$ edges

$$\textbf{[MST1]} \quad \min_{x} \sum_{(i,j)\in E} \phi_{ij} x_{ij}$$

$$s.t. \begin{cases} \sum_{(i,j)\in E} x_{ij} = n-1 \\ \sum_{(i,j)\in E(S)} x_{ij} \leq |S| - 1, \ \forall S \subset V, S \neq V, S \neq \emptyset \\ x_{ij} \in \{0,1\}, \ \forall (i,j) \in E \end{cases}$$

where $E(S) \subset E$ is a subset of edges with both ends in subset $S \subset V$. Constraint $\sum_{(i,j)\in E(S)} x_{ij} \leq |S| - 1$ ensures that there is no cycles in subset S.

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Minimum Spanning Tree: Cutset Formulation

- Cutset formulation is based on the fact that T is connected and has $n-1$ edges

$$\textbf{[MST2]} \quad \min_{x} \sum_{(i,j)\in E} \phi_{ij} x_{ij}$$

$$s.t. \begin{cases} \sum_{(i,j)\in E} x_{ij} = n-1 \\ \sum_{(i,j)\in \delta(S)} x_{ij} \geq 1, \ \forall S \subset V, S \neq V, S \neq \emptyset \\ x_{ij} \in \{0,1\}, \ \forall (i,j) \in E \end{cases}$$

where the cutset $\delta(S) \subset E$ is a subset of edges with one end in S and the other end in $V \setminus S$. Constraints $\sum_{(i,j)\in \delta(S)} x_{ij} \geq 1$ ensures that subsets S and $V \setminus S$ are connected.

Minimum Spanning Tree: Martin's formulation

$$\textbf{[MST4]} \quad \min_{x,y} \sum_{(i,j)\in E} \phi_{ij} x_{ij}$$

$$s.t. \begin{cases} \sum_{(i,j)\in E} x_{ij} = n-1 \\ y_{ij}^k + y_{ji}^k = x_{ij}, \ \forall (i,j)\in E, k \in V \\ \sum_{k\in V\setminus\{i,j\}} y_{ik}^j + x_{ij} = 1, \ \forall (i,j) \in E \\ x_{ij}, y_{ij}^k, y_{ji}^k \in \{0,1\}, \ \forall (i,j)\in E, k\in V \end{cases}$$

- $y_{ij}^k \in \{0,1\}$ denotes that edge (i,j) is in the spanning tree and vertex k is on the side of j
- The second constraint for $(i,j)\in E, k \in V$ guarantees that if $(i,j)\in E$ is selected into the tree ($x_{ij}=1$), any vertex $k \in V$ must be either on the side of j ($y_{ij}^k = 1$) or on the side of i ($y_{ji}^k = 1$). If $(i,j)\in E$ is not in the tree ($x_{ij}=0$), any vertex k cannot be on the side of j nor i ($y_{ij}^k = y_{ji}^k = 0$)
- The third constraint for $(i,j)\in E$ ensures that
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Minimum Spanning Forest

- Consider a graph G with m connected components
- Assume that the m connected components of G have vertex sets as $V_1, V_2 \cdots, V_m$
- Also assume E_i is the edge set induced by vertices in V_i from graph G
- Thus, each connected component of G can be considered as a subgraph $G_i = (V_i, E_i)$ of G.

Proposition

For the graph G with m connected components, denoted by $G_1, G_2, \cdots, G_m$, the forest F^*, consisting of spanning trees $T_1^*, T_2^*, \cdots, T_m^*$, is a minimum spanning forest of G if and only if each T_i^* is a minimum spanning tree for subgraph G_i $(i = 1, 2, \cdots, m)$. Furthermore, the number of edges in a spanning forest of G is $n - m$.

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Adapting Subtour Elimination and Cutset Formulations for MSF

Considering $S \subset V, S \neq \emptyset, S \neq V$, there are three cases for the subtour elimination constraints and cutset constraints:

(i) if $S \subset V_i$, $\sum_{(i,j)\in E(S)} x_{ij} \leq |S| - 1$; $\sum_{i\in S, j\in V\setminus S} x_{ij} \geq 1$;

(ii) if $S \subset V_{i_1} \cup V_{i_2} \cup \cdots \cup V_{i_k}$ $(2 \leq k \leq m)$ and $S \cap V_{i_1} \neq \emptyset, \cdots, S \cap V_{i_k} \neq \emptyset$, $\sum_{(i,j)\in E(S)} x_{ij} \leq |S| - k$; $\sum_{i\in S, j\in V\setminus S} x_{ij} \geq k$;

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Minimum Spanning Forest: Subtour Elimination Formulations

$$\textbf{[MSF1]} \quad \min \sum_{(i,j)\in E} \phi_{ij} x_{ij}$$

$$s.t. \sum_{(i,j)\in E} x_{ij} = n - m$$

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$$x_{ij} \in \{0,1\}, \ \forall (i,j) \in E$$

where the first constraint ensures that there are $n - m$ edges in the spanning forest.

Minimum Spanning Forest: Cutset Formulations

$$\textbf{[MSF2]} \quad \min \sum_{(i,j)\in E} \phi_{ij} x_{ij}$$

$$s.t. \sum_{(i,j)\in E} x_{ij} = n - m$$

$$\sum_{i\in S, j\in V\setminus S, (i,j)\in E} x_{ij} \geq \max_{i\in S, j\in V\setminus S} \mathbb{1}_{\{(i,j)\in E\}}, \ \forall S \subset V, S \neq V, S \neq \emptyset$$

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- IP formulations for MST and MSF

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- Polyhedral study and comparison of the formulations!

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