Consider the following web.

1) Give a ranking of the pages in the web where $x_k$ equal the number of backlinks for page $k$, i.e., the number of links into page $k$.

2) Explain why the rankings of pages 1 and 4 should not be the same since page 1 has a ranking from a more important page than page 4 does.

3) Now let $x_k$ be the ranking of page $k$ given so that the ranking of any page is equal to the sum of the rankings from each page linking into it. Show that this gives a systems of equations

$$
\begin{align*}
    x_1 &= x_3 + x_4 \\
    x_2 &= x_1 \\
    x_3 &= x_1 + x_2 + x_4 \\
    x_4 &= x_1 + x_2.
\end{align*}
$$

Now show that this system has no solutions! Hint: solve the last 3 equations and then show it is not consistent with the first equation.

4) Now let’s change the ranking so that each page gets a single vote that is divided equally among its outlinks, and the ranking $x_k$ is equal to the sum of
the votes times the rankings of all backlinks. The system is now

\[ x_1 = x_3 + \frac{1}{2} x_4 \]
\[ x_2 = \frac{1}{3} x_1 \]
\[ x_3 = \frac{1}{3} x_1 + \frac{1}{2} x_2 + \frac{1}{2} x_4 \]
\[ x_4 = \frac{1}{3} x_1 + \frac{1}{2} x_2. \]

Show that this system can be solved to get \( x_1 = 1, x_2 = \frac{1}{3}, x_3 = \frac{3}{4}, x_4 = \frac{1}{2} \).

Notice that \( x_1 \) has the highest ranking! This is because \( x_3 \) threw its whole vote to \( x_1 \) and so that even though \( x_3 \) got votes from three different sites, they still do not total as much as what \( x_1 \) gets. Note, usually we will rescale so that the sum is equal to 1, and so we get

\[ x_1 = \frac{12}{31}, \ x_2 = \frac{4}{31}, \ x_3 = \frac{9}{31}, \ x_4 = \frac{6}{31}. \]