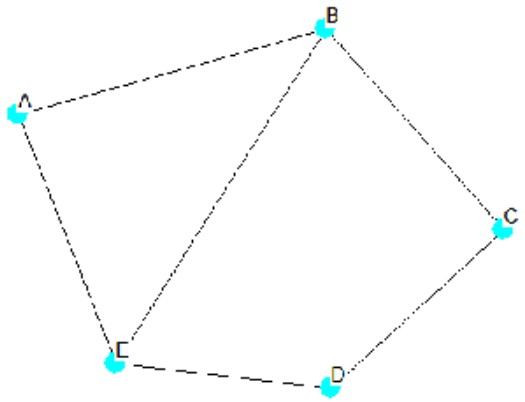
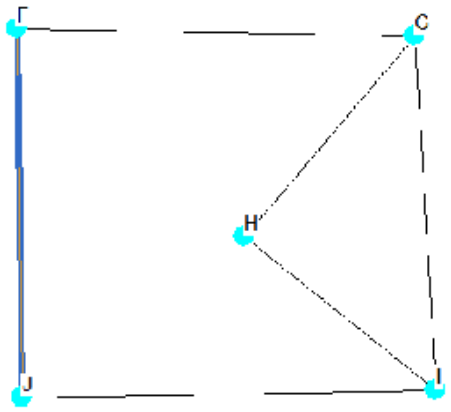
MATH 443/543

Test 1 Practice

Note: This sample test may not include all possible topics!

1. Consider the following two graphs.





Suppose ϕ is an isomorphism between the graph on the left and the graph on the right. For the following statements, say if they are possible, impossible, or there is not enough information. (Note, do not assume a is true when checking b, etc.)

a. $\phi(B) = F$

b. $\phi(B) = G$

c. $\phi(B) = H$

d. $\phi(A) = \phi(B)$

e. $\phi(C) = \phi(D)$

f. $\phi^{-1}(H) = A$

g. $\phi(BE) = GH$

h. $\phi(AB) = GH$

2.

a. Give an example of a graph which is hamiltonian but not eulerian. Justify your answer.
b. Give an example of a graph which is eulerian but not hamiltonian. Justify your answer.

3. Suppose G is a tree with p vertices. What is the maximal degree of a vertex?

4. A plane triangulation is a plane graph such that every region is bounded by exactly three edges (including the exterior region). Show that if G is a plane triangulation with p vertices

and q edges, then $q = 3p - 6$. (Hint: for each region add up the number of edges in its boundary and use Euler's theorem).

5. Give an example of a connected graph that is all of the following:

1. not planar,
2. 3-colorable, and
3. not 2-colorable.

Justify your answer (i.e., show it is not planar, is 3-colorable, is not 2-colorable).

6. Recall that $K_{n,n}$ denotes a complete bipartite graph. For which values of n is $K_{n,n}$ eulerian? For which values of n is $K_{n,n}$ hamiltonian? Explain.