1. Find the derivative of the following function:

\[ M(\alpha) = \tan^2 (2 + 3\alpha) \]

SOLUTION: Using the chain rule, we have

\[
\frac{dM}{d\alpha} = 2 \tan (2 + 3\alpha) \cdot 3 \cdot \frac{1}{\cos^2 (2 + 3\alpha)} = 6 \frac{\sin (2 + 3\alpha)}{\cos^3 (2 + 3\alpha)}
\]

2. Find the general solution to the following differential equation:

\[
\frac{dy}{dt} = k(y - A)
\]

Use separation of variables, integrate and you end up with

\[ y(t) = A + Ce^{kt} \]

where \( C \) is an arbitrary constant defined by your initial condition.
1. Find the derivative of the following function:

\[ g(x) = \tan^{-1}(3x^2 + 1) \]

**SOLUTION:** Need to remember that the general derivative for \( \arctan \) (a sigmoidal function) is

\[ \frac{d}{dx} \tan^{-1}(u) = \frac{1}{1 + u^2} \frac{du}{dx} \]

Using this, we have

\[ \frac{dg}{dx} = \frac{1}{1 + (3x^2 + 1)^2} \cdot 6x = \frac{6x}{9x^4 + 6x^2 + 2} \]

2. Find the general solution to the following differential equation:

\[ \frac{df}{d\theta} = ke^{-\theta/2} \]

Remember that the general solution for an ODE of the form

\[ \frac{df}{d\theta} = g(\theta) \]

is given by

\[ f(\theta) = \int g(\theta) \, d\theta \]

So computing the integral, we end up with the general solution

\[ f(\theta) = C - 2ke^{-\theta/2} \]

where \( C \) is an arbitrary constant defined by your initial condition.
Group 3

1. Find the derivative of the following function (assuming \(a\) and \(b\) are constants):

\[
g(u) = \frac{e^{au}}{a^2 + b^2}
\]

**NOTE:** also consider how your answer would change if you did not necessarily know if \(a\) or \(b\) were necessarily constant

**SOLUTION:** Given that most quantities are constants, we simply have

\[
\frac{dg}{du} = a \cdot \frac{e^{au}}{a^2 + b^2}
\]

However, if \(a\) and \(b\) were functions of \(u\), we would need to reapply the chain rule.

2. Show that any function of the form

\[
x = C_1 \cosh (\omega t) + C_2 \sinh (\omega t)
\]

satisfies the ODE

\[
x'' - \omega^2 x = 0.
\]

**Hint:** Can you express \(\cosh\) and \(\sinh\) in terms of functions you readily know how to take the derivative of?

Remember that

\[
\sinh (x) = \frac{e^x - e^{-x}}{2}
\]

and

\[
\cosh (x) = \frac{e^x + e^{-x}}{2}
\]

Thus one can readily show that \(\frac{d}{dx} \sinh (x) = \cosh (x)\) and \(\frac{d}{dx} \cosh (x) = \sinh (x)\). Thus

\[
\frac{d}{dt} (C_1 \cosh (\omega t) + C_2 \sinh (\omega t)) = \omega C_1 \sinh (\omega t) + \omega C_2 \cosh (\omega t)
\]

Carrying one step further and plugging back into the ODE, we see that the given function is indeed a solution.
Group 4

1. Find the derivative of the following function:

\[ s(y) = \sqrt[3]{(\cos^2(y) + 3 + \sin^2(y))} \]

SOLUTION: A bit tricky. Seems like we would need to use the chain rule. But note that the argument inside the cube root contains \( \cos^2(y) + 3 + \sin^2(y) \), which is just 1 no matter what the value of \( y \) is. Thus the problem is asking you to find the derivative of \( \sqrt[3]{1} \), which is just zero.

2. The birth rate of a population is proportional to its size \( x = x(t) \) with a constant of proportionality \( b > 0 \). The death rate of the population is proportional to the population size of a deadly virus \( y = y(t) \) with a constant of proportionality \( c > 0 \). The virus population has a negative per unit growth rate \( -r \).

a.) Write a differential equation and initial condition for the population sizes \( x = x(t) \) and \( y = y(t) \), assuming initial sizes \( x_o \) and \( y_o \).

b.) Show that the following are solutions to the ODEs you obtained in the previous part:

\[ x = \left( x_o - \frac{c}{r + b}y_o \right)e^{bt} + \frac{c}{r + b}y_o e^{-rt} \]

\[ y = y_o e^{-rt} \]

Translating the problem, we have

\[ \frac{dx}{dt} = bx - cy \]

\[ \frac{dy}{dt} = -ry \]

The second part of the problem merely requires one to take the derivative and do some algebra to show that the solution holds.
1. Find the derivative of the following function (assuming \(a\) and \(b\) are constants):

\[
w(r) = \frac{ar^2}{b + r^3}
\]

**SOLUTION:** Using the product (or quotient) and chain rules, we have

\[
\frac{dw}{dr} = \frac{2ar}{b + r^3} - \frac{ar^2}{(b + r^3)^2} \cdot 3r^2 = \frac{2abr - ar^4}{(b + r^3)^2}
\]

2. If one pound of fertilizer covers 250 square feet, estimate the amount of fertilizer needed to fertilize the fairway. (Note: Use \(n = 10\)) Make sure to clearly explain your answer!

![Figure 1: The width, in feet, at various points along the fairway of a hole on a golf course.](image)

The idea here is to approximate the area of the course using Riemann sums. Using the **LEFT** rule, we have \(\text{LEFT}(10) = (0 + 80 + 85 + 95 + 110 + 1 - 5 + 100 + 100 + 110) \times 100 = 89000\text{ sq. ft.}\). If one lbs. of fertilizer covers 250 sq. ft., then we would need 356 lbs. of fertilizer.