1. (10 points each) Consider $f(x) = x^4(13 - x)^3$
   a) Find $f'(x)$ and factor your answer completely.
   b) Find all critical points of $f(x)$. Use the first derivative test to determine local maxima and minima.

2. (15 points) Find the global maximum and minimum for the function on the closed interval
   $f(x) = xe^{-x^2/2}, -3 \leq x \leq 4$
3. Given $e^{\cos(\pi y)} = x^3 \arctan y$,
   
a) (10 points) Find $\frac{dy}{dx}$.
   
b) (5 points) Find local linearization of function $y = f(x)$ at $y = 1$. 

4. Given that \( f^{-1}(0) = 4 \) and \( f'(4) = 10 \),
   a)(10 points) find the equation of the tangent line of \( f^{-1} \) at \( x = 0 \).
   b)(5 points) find \( g'(0) \) where \( g(x) = \sinh x \ f^{-1}(x) \).

5. (10 points each) Let the one parameter family of functions \( g(x) = ax - 3x \ln(x) \), where \( x > 0 \).
   a) Find the \( x \)-coordinates of the critical points of \( g \).
   b) Use the second derivative test to classify the critical points.
6. (15 points) Find the **dimensions** of the box with largest possible volume made from 54 \( m^2 \) of material. The box has square base and a top.

7. (Bonus- 3 points) Is the following statement **True** or **False**? **Justify** or **give a counterexample**:

Let \( f \) be continuous on \((a, b]\) and differentiable on \((a, b)\) with \( f(a) = f(b) \), then there exists \( c \in (a, b) \) such that \( f'(c) = 0 \).