1. (a) (10 points each) Find the following integrals

   • \( \int \frac{t^5 - 1}{t} \, dt \)

   • \( \int_0^\pi (\cos x - e^x) \, dx \)

   • \( \int_{-6}^6 \sqrt{36 - x^2} \, dx \) (hint: use geometry).

(b) (10 points) Find the derivative \( \frac{d}{dx} \int_0^{x^2} \ln(8 + t^2) \, dt \).
2. (15 points) The length and width of a rectangle are changing. The length is increasing at 50 inches per hour and the width is decreasing at 3 inches per hour. How fast is the area of the rectangle changing when the length is 50 inches and the width is 118 inches?

3. (15 points) Find the solution of the initial value problem

\[ \frac{dy}{dx} = 3x + \frac{1}{\sqrt{x}}; \quad y(4) = 7. \]
4. (10 points) Find the positive constant \( c \) if the average value of the function \( v(x) = \frac{4}{x^2} \) on the interval \([1, c]\) is equal to 1.

5. (10 points each) Find the following limits

- \( \lim_{x \to 0^+} x \ln x \).

- \( \lim_{t \to 0^+} \left( \frac{1}{7t} - \frac{1}{e^t - 1} \right) \)