(1) For any $\omega > 0$, let
\[ \Gamma(\omega) := \int_{0}^{\infty} x^{\omega-1} e^{-x} dx. \]
Show that if $\omega$ is a positive integer then $\Gamma(\omega) = (\omega - 1)!$

(2) Find the mean and variance of the Gamma($\lambda, \omega$) distribution.

(3) Suppose $X$ is an exponential random variable with parameter $\lambda = 1$. Find the distribution and density functions of $Y = \ln(X)$.
\textbf{Note:} This is called the double exponential distribution.

(4) Suppose $X \sim N(0,1)$. Use integration by parts to show that $\mathbb{E}(X^k) = (k-1)\mathbb{E}(X^{k-2})$.
Derive that $\mathbb{E}(X^k) = 0$ for all odd $k \geq 1$. Compute $\mathbb{E}(X^4)$ and $\mathbb{E}(X^6)$. Derive a general formula for $\mathbb{E}(X^{2k})$. 