Test 1 **Introduction to Linear Algebra** **May 29, 2018**

MA 313

Name: ______________________ **Signature:** ______________________

SHOW ALL YOUR WORK!

1. [10 points] Row reduce each of the following matrices into the **row reduced echolon form**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

2. [10 points] Let

$$\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 1 & 4 \\ 2 & -3 & 2 \\ 4 & -8 & 12 \end{bmatrix}.$$

Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A?

3. [10 points] Find the value(s) of h for which the vectors

$$\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ h \end{bmatrix}$$

are linearly independent.

4. [20 points] Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined as

$$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$$

for every real numbers $x_1, x_2 \in \mathbb{R}$.

(a) Find the standard matrix A of the linear transformation T.

(b) Find $\mathbf{x}$ such that $T\mathbf{x} = (-1, 4, 9)$.

(c) Is T one-to-one? onto? Explain!

5. [20 points] Let A be a 3×4 matrix. Suppose that the third (last) row in the row reduced echolon form of A is the vector $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$.

   (a) Describe geometrically the solution set of the homogeneous matrix equation $A\mathbf{x} = \mathbf{0}$.

   (b) Find a particular solution of the matrix equation $A\mathbf{x} = \mathbf{a}_2$, where $\mathbf{a}_2$ is the second column of A.

   (c) Describe geometrically the solution set of $A\mathbf{x} = \mathbf{a}_2$.

   (d) Determine whether the columns of A are linearly independent, Explain.

6. [10 points] Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in **parametric vector form**, where A is row equivalent to the following

$$\begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

7. [20 points] Mark each statement as **True** or **False**.

- (   ) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effects on the columns of the $n \times n$ identity matrix.
- (   ) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\mathbf{x}$ in $\mathbb{R}^n$ is mapped onto some vectors in $\mathbb{R}^m$.
- (   ) If A is 3×2 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot be one-to-one.
- (   ) Every matrix is row equivalent to a unique echelon form.
- (   ) The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.
- (   ) If A is an $m \times n$ matrix and $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^m$ then A has m pivot columns.
- (   ) If an $n \times n$ matrix A has n pivot positions then the reduced echelon form of A is the $n \times n$ identity matrix.
- (   ) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ are linearly independent then $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are not in $\mathbb{R}^2$.
- (   ) If none of the vectors in the set $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ in $\mathbb{R}^3$ is a multiple of one of the other vectors then S is linearly independent.
- (   ) The columns of any 4×3 matrix are linearly independent.
- (   ) If A is 2×5 matrix and T is the matrix transformation $T\mathbf{x} = A\mathbf{x}$, then the domain of T is $\mathbb{R}^5$ and its range is subset of $\mathbb{R}^2$.